Arrested Development: Theory and Evidence of Supply-Side Speculation in the Housing Market

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Abstract

This paper studies the role of disagreement in amplifying housing cycles. Speculation is easier in the land market than in the housing market due to frictions that make renting less efficient than owner-occupancy. As a result, undeveloped land both facilitates construction and intensifies the speculation that causes booms and busts in house prices. This observation reverses the standard intuition that cities where construction is easier experience smaller house price booms. It also can explain why the largest house price booms in the United States between 2000 and 2006 occurred in areas with elastic housing supply.

JEL Codes: D84, G12, G14, R31

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Asset prices go through periods of sustained price increases, followed by busts. To explain these episodes, economists have developed theories based on disagreement, speculation, and strategic trading. This literature focuses on the behavior of asset prices in stock markets, but it is natural to ask whether these ideas can explain housing markets as well. Like any other financial asset, housing is a traded, durable claim on uncertain cash flows. An enduring feature of housing markets is booms and busts in prices that coincide with widespread disagreement about fundamentals (Shiller, 2005), and there is a long history of investors using real estate to speculate about the economy (Kindleberger, 1978; Glaeser, 2013).

This paper incorporates disagreement into a housing model to examine whether the insight that disagreement raises stock prices generalizes to the housing market. Housing differs in two fundamental ways from the typical asset studied in finance. First, the typical asset in financial models is in fixed supply. In contrast, elastic supply is central to housing markets, as firms respond to high prices with new construction (Gyourko, 2009). Second, while the typical financial asset pays cash dividends, the dividend paid by housing is the flow utility enjoyed by end users. This flow utility has different values for different people and is not perfectly transferable, leading many people to prefer owning to renting (Henderson and Ioannides, 1983).

We study a two-period model of a housing market with two classes of agents. Potential residents receive heterogeneous utility from consuming housing that accrues only when they own their houses. Developers supply housing in a competitive market, buying land at market prices and converting it into housing for a constant resource cost. As in Saiz (2010), the amount of developable land is fixed due to geographic and regulatory constraints. At \( t = 0 \), there is an initial number of potential residents \( N_0 \), and at \( t = 1 \) the number of potential residents grows to \( N_1 = e^{\mu x} N_0 \), where \( x \) is a positive shock. The larger is \( N_1 \), the greater the price of land and housing at \( t = 1 \). At \( t = 0 \), agents all observe the shock \( x \), but they “agree to disagree” about \( \mu \). In this context, we define a “house price boom” as the reaction of the house price at \( t = 0 \) to the shock \( x \).

Our results characterize how the size of the house price boom varies with \( N_0 \). When \( N_0 \) is very small, there is no house price boom because the price of land remains equal to 0. When \( N_0 \) is very large, initial housing demand can be so strong that at \( t = 0 \) all space is held by homeowners. In this case, the house price at \( t = 0 \) reflects both heterogeneous expectations about the \( t = 1 \) price and homeowners’ flow utility for housing. The latter

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1Beginning with Miller (1977), a large literature has used models of disagreement to explain asset pricing patterns in the stock market. Hong and Stein (2007) survey this literature, which includes Harrison and Kreps (1978), Morris (1996), Diether, Malloy and Scherbina (2002), Scheinkman and Xiong (2003), Hong, Scheinkman and Xiong (2006), and Simsek (2013). Other papers have applied speculative finance models to housing; see, for example, Piazzesi and Schneider (2009), Favara and Song (2014), Giglio, Maggiori and Stroebel (2014), and Burnside, Eichenbaum and Rebelo (2015). Unlike those papers, our work focuses on housing supply.
mitigates the impact of the former. Finally, when $N_0$ is intermediate, some land at $t = 0$ is left unoccupied by homeowners and is held by developers. These *supply-side speculators* hold land only when they have optimistic beliefs about $\mu$, and it is these optimistic beliefs that drive prices. Consequently, prices at $t = 0$ are most sensitive to disagreement and optimism for cities at an intermediate level of development. For the same reason, the house price at $t = 0$ is most sensitive to $x$, i.e., booms are larger, in these cities.

Stated in terms of available land in the city, the house price boom is largest for intermediate values of initial land supply. This non-monotonicity between the house price boom and supply contrasts with prior work on disagreement—which does not consider the unique aspects of housing—and prior work on housing cycles—which does not consider disagreement. Taken separately, each approach predicts a monotonically declining relationship between a house price boom and initial land supply (Hong, Scheinkman and Xiong, 2006; Glaeser, Gyourko and Saiz, 2008; Paciorek, 2013). By joining these approaches, we provide a new insight about housing cycles that neither offers alone.

We demonstrate the robustness of this result in several extensions that relax the model’s assumptions in different ways. First, we consider the case where developers can issue equity and investors can short-sell that equity. Second, we consider an extension in which landlords can speculate in the housing market and rent out housing to pessimistic residents. In a final extension, we generalize the model to the case in which the supply elasticity declines continuously with the level of initial demand. We also formally show how disagreement reduces welfare (in the sense of Brunnermeier, Simsek and Xiong, 2014) by reallocating space from high-flow-utility pessimists to low-flow-utility optimists and developers.

The model’s core insight helps explain the variation in 2000-2006 house price booms across US cities. As shown by Davidoff (2013), a static supply-demand framework with a common national demand shock cannot account for cities in the “sand states” of Arizona, California, Florida, and Nevada that experienced both strong price and quantity growth. One possibility is that the demand shocks in these cities were especially large due to local credit conditions, differences in productivity and amenities, or heightened speculative activity by homebuyers (Barlevy and Fisher, 2011; Davidoff, 2013; Gao, Sockin and Xiong, 2016). Our model offers an alternative explanation: house prices were more sensitive to a demand shock in these cities because the cities were at an intermediate level of development, and market participants disagreed about future prices.

We offer several pieces of empirical evidence in support of this explanation. Land price

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2Simsek (2013) predicts that the introduction of new non-redundant financial assets increases the “speculative variance” of portfolio values, but this result does not apply to our findings because the price of undeveloped land is perfectly correlated with the price of housing in our equilibrium.

3Understanding the factors driving these outlier cities is crucial for evaluating research instrumenting for price growth with supply elasticity. Davidoff (2016) discusses problems with this instrument.
growth from 2000 to 2006 was high and closely correlated with house price growth across cities, whereas construction cost growth was not. Matching this fact distinguishes our model from Gao, Sockin and Xiong (2015), who offer a theory of non-monotonic house price booms that is agnostic on the differential roles of land prices and construction costs. We also document land market speculation from US public homebuilders. These firms amassed land far in excess of their immediate construction needs during the boom, while investors short-sold homebuilder stocks more than nearly every other industry. The model predicts these outcomes for developers in intermediate cities only in the presence of disagreement.

Although we do not systematically examine land constraints in sand state cities, the case of Las Vegas offers a stark illustration of our model. Las Vegas is surrounded by land owned by the federal government, and Congress passed a law in 1998 prohibiting the sale of land outside a development ring depicted in Figure 1. Some people in 2007 believed that the remaining land would be fully developed by 2017 (McKinley and Palmer, 2007). During the boom, land prices rose from $150,000 to $700,000 per acre before losing the gains in the bust. In a striking example of supply-side speculation, a single land development fund, Focus Property Group, won the majority of the large land parcels auctioned between 2002 and 2005 by the federal government in Las Vegas, obtaining 5% of the undeveloped land remaining within the barrier.4

The paper proceeds as follows. Section 1 presents the core model. Sections 2 and 3 characterize equilibrium with agreement and disagreement, respectively. Section 4 presents extensions of the core model. Section 5 analyzes the 2000-2006 US housing boom and subsequent bust. Section 6 concludes.

1 A Housing Market with Disagreement

Housing Supply and Developer Demand. The city we study has a fixed amount of space $S$. At the beginning of period 0, all of this space exists as undeveloped land that can be used for housing. Housing and land trade in spot markets each period but cannot be sold short.5 The price of land and housing at $t$ are $p^l_t$ and $p^h_t$, respectively.

Developers are private firms endowed with the entire supply of land at the beginning of period 0. Developers can borrow or lend freely in global capital markets at an interest rate normalized to 0. In each period, a developer makes three decisions: how much land to buy or sell, how much housing to build, and how much housing to sell. Building a unit of housing

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4The sources for this paragraph include price data from Applied Analysis, auction records from the Bureau of Land Management, news reports in the Las Vegas Review Journal, and land-use information from the Southern Nevada Regional Planning Commission.

5Short-sale constraints in the housing market result from a lack of asset interchangeability. Although housing is homogeneous in the model, empirical housing markets involve large variation in characteristics across houses. This variation in characteristics makes it essentially impossible to cover a short.
requires one unit of land and a resource cost $k_t$. To simplify equations and keep the user cost of housing constant over time, we set $k_0 = 2k$ and $k_1 = k$ for some $k > 0$. At $t = 1$, the owners of each developer receive the proceeds from liquidation.

Each developer maximizes its subjective expectation of its liquidation value. Denote the holdings of housing, land, and bonds at the beginning of $t$ by $H_t$, $L_t$, and $B_t$, respectively. Denote the control variables of home sales, land purchases, and home construction at each $t$ by $H^\text{sell}_t$, $L^\text{buy}_t$, and $H^\text{build}_t$, respectively. At $t = 1$, the liquidation value $\pi$ of a developer is the outcome of the constrained optimization problem:

$$\pi(p^h_1, p^l_1, H_1, L_1, B_1) = \max_{H^\text{sell}_1, L^\text{buy}_1, H^\text{build}_1} p^h_1 H^\text{sell}_1 - p^l_1 L^\text{buy}_1 - k H^\text{build}_1 + B_1$$

subject to

- $H^\text{sell}_1 \leq H_1 + H^\text{build}_1$
- $H^\text{build}_1 \leq L_1 + L^\text{buy}_1$.

The actions $(H^\text{sell}_1)^*$, $(L^\text{buy}_1)^*$, and $(H^\text{build}_1)^*$ chosen by the developer maximize this problem. At $t = 0$, the developer maximizes its subjective expectation of this liquidation value:

$$(H^\text{sell}_0)^*, (L^\text{buy}_0)^*, (H^\text{build}_0)^* \in \arg \max_{H^\text{sell}_0, L^\text{buy}_0, H^\text{build}_0} \mathbb{E}\pi(p^h_1, p^l_1, H_1, L_1, B_1)$$

subject to

- $H^\text{sell}_0 \leq H^\text{build}_0$
- $H^\text{build}_0 \leq L_0 + L^\text{buy}_0$
- $H_1 = H^\text{build}_0 - H^\text{sell}_0$
- $L_1 = L_0 + L^\text{buy}_0 - H^\text{build}_0$
- $B_1 = p^h_0 H^\text{sell}_0 - p^l_0 L^\text{buy}_0 - 2k H^\text{build}_0$.

At $t = 0$, developers may differ only in their land endowments $L_0$ and in their beliefs about $p^h_1$ and $p^l_1$ (as specified below). The sum of the land endowments across developers equals $S$. Developers take prices as given, which is consistent with evidence we discuss in Section 5 on perfect competition in the homebuilding industry.

**Individual Housing Demand.** Potential residents derive utility from consumption and from owning and occupying housing. There are two disjoint groups of potential residents: one arriving at $t = 0$ and one arriving at $t = 1$. Upon arrival, each potential resident decides whether to buy a house. Utility comes from consumption $c$ at $t = 1$ and any housing services $v$ received in the period of arrival. Utility is linear and separable in housing and consumption: $u = c + v$ if the potential resident owns a house in the period of her arrival.

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6We abstract from the possibility of overbuilding by allowing developers to build negative amounts of housing by recouping $k_t$ from turning a house into land.
and $u = c$ otherwise.

A potential resident who buys at $t = 0$ decides whether to sell her house at $t = 1$. As with developers, potential residents at $t = 0$ may borrow in global capital markets at an interest rate of $0$. Denote the control variables of whether or not to buy or sell a house by $H_t^{buy}$ and $H_t^{sell}$, restricting these to equal 0 or 1. At $t = 1$, an arriving potential resident chooses

$$(H_1^{buy})^* \in \arg \max_{H_1^{buy}} H_1^{buy}(v - p_1^h)$$

subject to $H_1^{buy} \in \{0, 1\}$,

and the utility of potential residents who bought at $t = 0$ equals

$$u(p_1^h, B_1, v) = \max_{H_1^{sell}} H_1^{sell} p_1^h + B_1 + v$$

subject to $H_1^{sell} \in \{0, 1\}$,

where the choice $(H_1^{sell})^*$ maximizes this problem. At $t = 0$, arriving potential residents maximize the subjective expectation of their utility:

$$(H_0^{buy})^* \in \arg \max_{H_0^{buy}} H_0^{buy} E u(p_1^h, B_1, v)$$

subject to $H_0^{buy} \in \{0, 1\}$

$$B_1 = -p_0^h H_0^{buy}.$$ 

At $t = 0$, potential residents may differ only in their housing utility $v$ and in their beliefs about $p_1^h$ (as specified below). At $t = 1$, arriving potential residents may differ only in $v$.

Denote by $D(v)$ the complementary cumulative distribution function (1 minus the CDF) of $v$ among arriving potential residents. $D(v)$ is a time-invariant function that encodes heterogeneity in housing flow utility. We make the following functional form assumption about $D(v)$:

**Assumption 1.** There exists $\epsilon > 0$ such that

$$D(v) = \begin{cases} 
1 & \text{if } v < k \\
(k/v)\epsilon & \text{if } v \geq k.
\end{cases}$$

By Assumption 1, no potential residents have housing utility $v$ less than $k$. This restriction implies all potential residents are willing to purchase housing at cost. As a result, no residents buy housing only because of expected capital gains. In the model, such pure speculators
would instead be classified as developers. Assumption 1 also invokes a constant elasticity of demand for housing, which allows us to derive simple analytic results.

Our utility specification makes two implicit assumptions about resident behavior. First, because utility is separable and linear in $c$, potential residents are risk-neutral. As a result, the purchase decisions of potential residents at $t = 0$ are not affected by the type of hedging motives studied by Piazzesi, Schneider and Tuzel (2007). Second, because potential residents receive utility from only one house, their housing utility displays diminishing marginal returns. This property leads homeownership to be dispersed among residents in equilibrium.

**Aggregate Demand and Beliefs.** Aggregate resident demand for housing depends on the number of potential residents and the joint distribution of housing utility and beliefs. The number of arriving potential residents at $t$ equals $N_tS$, where $N_t > 0$. The growth in $N_t$ between $t = 0$ and $t = 1$ is given by

$$\log(N_1/N_0) = \mu^{true}x,$$

where $x \geq 0$ is a shock and $\mu^{true}$ is some constant. At $t = 0$, all agents observe $N_0$ and $x$. They do not observe $\mu^{true}$, the data needed to map the information shock $x$ to the demand growth rate. Agents learn the value of $\mu^{true}$ at $t = 1$. The resolution of uncertainty at $t = 1$ is common knowledge at $t = 0$.

At $t = 0$, agents may disagree about the value of $\mu^{true}$. Agent beliefs at $t = 0$ are indexed by $\theta \in \Theta \subset \mathbb{R}$. An agent of type $\theta$ believes with certainty that $\mu^{true} = \mu(\theta)$, where $\mu : \Theta \rightarrow \mathbb{R}$ is a weakly increasing function. When $\mu(\cdot)$ is not constant, beliefs vary across residents, and knowing the beliefs of other residents does not lead to any Bayesian updating. This “agree-to-disagree” assumption rules out any inference from prices at $t = 0$. Therefore, for instance, a developer who holds land in equilibrium can realize that it is the most optimistic developer, but this realization fails to change the developer’s belief.

As argued by Morris (1996), this heterogeneous prior assumption is most appropriate when investors face an unusual, unexpected situation like the arrival of the shock we are studying. Housing booms historically accompany unanticipated events like the settlement of new cities or the discovery of new resources (Glaeser, 2013). In the case of the US housing boom between 2000 and 2006, Mian and Sufi (2009) suggest that the shock was the arrival of new securitization technologies that expanded credit to homebuyers, although an equally

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7We explore the interaction between speculative and fundamental motives in related work (DeFusco, Nathanson and Zwick, 2016).

8This dispersion is partly due to the limitation that potential residents can buy at most one house. Section 4 considers an extension in which potential residents can buy unlimited amounts of housing and rent it to tenants.
valid interpretation would be demographic shifts leading to a secular increase in housing demand. The shock to housing demand between 2000 and 2006 is $x$, and $\mu^{true}$ represents the degree to which this shock persists after 2006. Even economists disagreed about $\mu^{true}$ during the boom (Gerardi, Foote and Willen, 2010).

Denote by $f_d$ and $f_r$ the distribution of $\theta$ across developers and residents, respectively. We allow these distributions to differ in order to study the equilibrium effects of developer and resident beliefs separately. We make two key assumptions about these distributions:

**Assumption 2.** $\theta$ and $\nu$ are independent for potential residents.

**Assumption 3.** $\theta_j^{max} \equiv \max \text{supp } f_j$ exists for $j \in \{d, r\}$.

Assumption 2 guarantees that the two sources of heterogeneity among potential residents at $t = 0$—their beliefs and their housing utility—are independent from one another. Assumption 3 guarantees the existence of a most optimistic developer. Given that developers can access unlimited quantities of financing, an equilibrium would not exist without this regularity condition. We define $\mu_j^{max} \equiv \mu(\theta_j^{max})$ for $j \in \{d, r\}$.

To study the marginal effects of disagreement on equilibrium, we adopt the specification

$$\mu(\theta) = \overline{\mu} + z\theta,$$

where $\overline{\mu}, z \geq 0$. When $z = 0$, all agents agree that $\mu^{true} = \overline{\mu}$. The following assumption guarantees the existence in each class of agents of optimists and pessimists relative to the agreement benchmark when $z > 0$:

**Assumption 4.** $\int_{\theta < 0} f_j(\theta) d\theta > 0$ and $\int_{\theta > 0} f_j(\theta) d\theta > 0$ for $j \in \{d, r\}$.

Given Assumption 4, larger values of $z$ lead to greater variation in beliefs. The assumption is sufficiently general that an increase to $z$ may also alter the mean beliefs among each group of agents.

**Land and Housing Market Equilibrium.** In an equilibrium, a city is **constrained** if all space is used for housing, **unconstrained** if some space remains as land and the price of land is 0, and **intermediate** if some space remains as land but the price of land is positive. This classification partitions all equilibrium outcomes and will prove useful for describing them.

The land market clears at $t$ if the sum of $(L_t^{buy})^*$ across developers equals 0, and the housing market clears at $t$ when the sum of $(H_t^{sell})^*$ across developers and potential residents equals the sum of $(H_t^{buy})^*$ across potential residents. The prices $p^1_t$ and $p^h_t$ constitute

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9 Although there exist theories of why consumer beliefs may be driven by preferences (Bénabou and Tirole, 2016), independence of beliefs and preferences seems like a reasonable starting point.

10 The restriction $\overline{\mu} \geq 0$ implies that agents expect a non-negative growth rate without disagreement and simplifies our analysis by allowing us to focus on the case of a positive shock to the housing market.
an equilibrium when the land and housing markets clear at $t = 1$. The following lemma characterizes this equilibrium:

**Lemma 1.** Given $N_1$, a unique equilibrium at $t = 1$ exists and is given by

$$p^h_1 = \begin{cases} 
  k & \text{if } N_1 < 1 \quad \text{(unconstrained)} \\
  kN_1^{1/\epsilon} & \text{if } N_1 \geq 1 \quad \text{(constrained)}
\end{cases}$$

and $p^l_1 = p^h_1 - k$.

Denote the prices in this equilibrium by $p^l_1(N_1)$ and $p^h_1(N_1)$.

These simple expressions for equilibrium prices depend on the model’s assumptions in the following way. Because construction is reversible, $p^h_1 = p^l_1 + k$ and initial conditions such as endowments and the housing stock are irrelevant for prices. Thus only $N_1$, the number of arriving potential residents, matters for prices at $t = 1$. If $N_1 < 1$, then the available space $S$ exceeds the number of potential residents who want to buy at $p^h_1 = k$. As a result, land is free and $p^h_1 = k$. If $N_1 > 1$, then more potential residents want to buy at $p^h_1 = k$ than there is available space, so the price of housing rises. In this case, because the elasticity of $D(\cdot)$ for $v \geq k$ is assumed to be $\epsilon$, the pass-through of more potential residents to the house price equals $1/\epsilon$, giving the formula in Lemma 1.

At $t = 0$, an agent of type $\theta$ believes with certainty that house and land prices at $t = 1$ will equal $p^h_1(e^{\mu(\theta)x}N_0)$ and $p^l_1(e^{\mu(\theta)x}N_0)$, respectively. Given these beliefs, $p^l_0$ and $p^h_0$ constitute an equilibrium when the land and housing markets clear at $t = 0$. The following lemma characterizes this equilibrium:

**Lemma 2.** Given $N_0$, $x$, and $z$, a unique equilibrium at $t = 0$ exists, and in this equilibrium $p^l_0 = p^h_0 - 2k$.

Denote prices in this equilibrium by $p^l_0(N_0, x, z)$ and $p^h_0(N_0, x, z)$. Sections 2 and 3 fully characterize these prices in the cases of agreement and disagreement, respectively.

As above, the result that $p^l_0 = p^h_0 - 2k$ follows from the reversibility of construction. If $p^l_0 < p^h_0 - 2k$, then developers would want to buy an infinite amount of land and build houses to sell; if $p^l_0 > p^h_0 - 2k$, developers would want to buy an infinite amount of housing to revert to land. Markets would not clear in either case, so neither inequality can hold in equilibrium.

## 2 Equilibrium with Agreement

In this section, we describe the equilibrium house price at $t = 0$ under agreement. This special case of the model, in which $z = 0$, provides the baseline to which we compare the
equilibrium under disagreement, in which $z > 0$. Proposition 1 characterizes the equilibrium at $t = 0$ for land holdings, the price of housing, and the effect of the shock $x$ on the price of housing, which we call the house price boom.

**Proposition 1.** In equilibrium, when $z = 0$ developers hold land at $t = 0$ if and only if $N_0 < 1$. The house price at $t = 0$ equals

$$p^h_0(N_0, x, 0) = \begin{cases} 
2k & \text{if } N_0 \leq e^{-\frac{\mu x}{\epsilon}} \text{ (unconstrained)} \\
 k + ke^{\frac{\mu x}{\epsilon}N_0^{1/\epsilon}} & \text{if } e^{-\frac{\mu x}{\epsilon}} < N_0 < 1 \text{ (intermediate)} \\
k(1 + e^{\frac{\mu x}{\epsilon}}N_0^{1/\epsilon}) & \text{if } N_0 \geq 1 \text{ (constrained)}
\end{cases}$$

and the house price boom

$$\frac{p^h_0(N_0, x, 0)}{p^h_0(N_0, 0, 0)} - 1 = \begin{cases} 
0 & \text{if } N_0 \leq e^{-\frac{\mu x}{\epsilon}} \text{ (unconstrained)} \\
\frac{1}{2}(e^{\frac{\mu x}{\epsilon}N_0^{1/\epsilon}} - 1) & \text{if } e^{-\frac{\mu x}{\epsilon}} < N_0 < 1 \text{ (intermediate)} \\
\frac{1}{2}(e^{\frac{\mu x}{\epsilon}} - 1) & \text{if } N_0 \geq 1 \text{ (constrained)}
\end{cases}$$

weakly increases in $N_0$.

The price at $t = 0$ consists of two terms: one that reflects the housing utility for the marginal buyer today, and one that reflects the common expectation of this marginal utility tomorrow. Using Lemma 1, we may write $p^h_0(N_0, x, 0) = p^h_1(N_0) + p^h_1(e^{\frac{\mu x}{\epsilon}N_0})$. When $N_0 \leq e^{-\frac{\mu x}{\epsilon}}$, agents expect that $N_1 \leq 1$ and that $p^h_1 = k$, leading to a price today of $2k$. In the intermediate case when $e^{-\frac{\mu x}{\epsilon}} < N_0 < 1$, land is available today but agents agree it will not be tomorrow. When $N_0 \geq 1$, housing is constrained both today and tomorrow.

Under agreement, the house price boom rises monotonically in the level $N_0$ of demand at $t = 0$. When demand is low, the shock fails to raise prices because agents continue to expect the city will be unconstrained at $t = 1$. Cities at an intermediate level of demand experience intermediate booms, with larger booms in places with more initial demand. Over the range $e^{-\frac{\mu x}{\epsilon}} < N_0 < 1$, a larger $N_0$ indicates that the city is closer to being constrained so that a greater share of the shock $x$ appears in prices at $t = 1$. When demand is sufficiently high ($N_0 \geq 1$), the shock passes through at a constant rate to the price of housing. Pass-through does not vary with $N_0$ over this range because of the constant elasticity specification of $D(v)$ in Assumption 1.

### 3 Equilibrium with Disagreement

This section describes the equilibrium at $t = 0$ under disagreement about future demand growth, which holds when $x, z > 0$ as assumed throughout this section. We study the effect...
of disagreement on land holdings and the price of housing, the aggregation of beliefs into the price of housing, and the variation in house price booms across cities depending on their initial level of development.

3.1 Dispersed Homeownership and Land Speculation

Proposition 2 formally describes the equilibrium allocation of land and housing at \( t = 0 \).

**Proposition 2.** In equilibrium, housing is held at \( t = 0 \) by potential residents of each type \( \theta \in \text{supp } f_r \). For land holdings, there exists \( N_0^* (x, z) \in \mathbb{R}_{>1} \cup \{\infty\} \) such that:

- (Unconstrained) If \( N_0 \leq e^{-\mu_{d}^{\text{max}}x} \), then some developers hold land at \( t = 0 \), and these developers may be of any type \( \theta \in \text{supp } f_d \).

- (Intermediate) If \( e^{-\mu_{d}^{\text{max}}x} < N_0 < N_0^* (x, z) \), then some developers hold land at \( t = 0 \), and all of these developers have type \( \theta_{\text{max}} \). Furthermore, there exists \( L^* \in (0, S) \) such that if the sum of \( L_0 \) across developers for whom \( \theta = \theta_{\text{max}} \) is less than \( L^* \), then the sum of \( (L_0^{\text{buy}})^* \) across these developers exceeds the sum of \( (H_0^{\text{build}})^* \) across them.

- (Constrained) If \( N_0 \geq N_0^* (x, z) \), then no developers hold land at \( t = 0 \).

\( N_0^* (x, z) = \infty \) if and only if \( \int_{\theta \geq \theta_{\text{max}}}^{} f_r (\theta) d\theta = 0 \) and \( \int_{\theta < \theta_{\text{max}}}^{} (e^{\mu_{d}^{\text{max}}x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon} f_r (\theta) d\theta \leq 1 \).

In equilibrium, homeownership at \( t = 0 \) is dispersed among potential residents of all beliefs. A potential resident buys a house at \( t = 0 \) if \( v > p_{h}^0 - E p_{h}^1 \). The number of homebuyers of type \( \theta \) equals \( N_0 SD (p_h^0 (N_0, x, z) - p_h^1 (e^{\mu(\theta)x} N_0)) \), which is positive for all \( \theta \in \Theta \). Positivity depends on Assumption 1, which guarantees that \( D(v) > 0 \) for any argument. There exist potential residents with arbitrarily high flow utility, so no matter how expensive housing appears to them, some potential residents of each type choose to buy.\(^{11}\)

Developers choose not to hold housing at the end of \( t = 0 \) because it is cheaper to hold land and build a house at \( t = 1 \) for \( k \) instead of paying \( 2k \) at \( t = 0 \). In the land market, a developer wants to purchase an infinite amount of land if \( p_0^l < E p_1^l \). This situation cannot hold in equilibrium, so for all \( \theta \in \text{supp } f_d \),

\[ p_0^l (N_0, x, z) \geq p_1^l (e^{\mu(\theta)x} N_0). \quad (1) \]

For \( \theta \) such that (1) holds with equality, a developer of type \( \theta \) is indifferent to holding land at \( t = 0 \). For \( \theta \) such that (1) is a strict inequality, a developer of type \( \theta \) chooses not to

\(^{11}\)This point relates to the work of Cheng, Raina and Xiong (2014), who find that securitized finance managers did not sell off their personal housing assets during the boom. They interpret this result as evidence that these managers had the same beliefs as the rest of the market about future house prices. An alternative interpretation is that the managers did doubt market valuations, but continued to own housing because they derived sufficiently high utility from housing to compensate for low expected capital gains.
hold land at the end of $t = 0$, either by selling it or by building houses using the land and selling the houses. As a result, only developers for whom $p^l_1(e^{\mu(\theta)x}N_0) = p^l_1(e^{\mu_{\text{max}}x}N_0)$ may hold land at $t = 0$. This is a simple statement of the result that prices in asset markets with disagreement and limited short-selling tend to reflect the beliefs of the most optimistic agents (Miller, 1977).

Under two conditions, undeveloped land remains at the end of $t = 0$ and is held only by developers for whom $\theta = \theta_{d}^{\text{max}}$. First, developers must disagree about their expectations of $p^l_1$ so that $p^l_1(e^{\mu(\theta)x}N_0) < p^l_1(e^{\mu_{\text{max}}x}N_0)$ when $\theta < \theta_{d}^{\text{max}}$. Because $p^l_1(\cdot)$ strictly increases only on $[1, \infty)$, this monotonicity condition holds if and only if $N_0 > e^{-\mu_{d}^{\text{max}}x}$. The second condition is that some undeveloped land remains at the end of $t = 0$. This condition is met if $S$ exceeds potential resident demand at the price that attracts optimistic developers to hold land:

$$S > N_0S \int_\Theta D(k + p^h_1(e^{\mu_{d}^{\text{max}}x}N_0) - p^h_1(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta.$$ 

The proof of Proposition 2 shows there exists a cutoff $N_0^*(x, z) > 1$ such that the above inequality holds if and only if $N_0 < N_0^*(x, z)$. When $N_0^*(x, z) < \infty$, a sufficiently large number of potential residents will always outbid the most optimistic developers for space.

Proposition 2 shows how the housing and land markets differ in the concentration of ownership among optimists. While potential residents of all beliefs own housing, land is owned only by the most optimistic developers in cities at which the initial level of demand takes on intermediate values. The idea that real estate speculation transpires largely in land markets departs from the literature, which has focused mostly on investors in houses.\(^\text{12}\)

Developers can carry land over between $t = 0$ and $t = 1$ and thus care about future prices. This feature raises the possibility they buy land in advance of their immediate construction needs according to their beliefs about future demand. We refer to this behavior as supply-side speculation. In the model, if undeveloped land remains and the most optimistic developers own all of it, then they must have bought more than they used for homebuilding (unless they were initially endowed with enough land). Proposition 2 formally states this prediction, which we explore in Section 5 by examining the balance sheets of US public homebuilders during the boom and bust of the early 2000s.

### 3.2 Belief Aggregation

We next characterize how the equilibrium house price at $t = 0$ aggregates the disparate beliefs of developers and potential residents. For $N_0 \geq N_0^*(x, z)$, we define the aggregate

\(^{12}\text{See, for example, Barlevy and Fisher (2011), Haughwout et al. (2011), Bayer et al. (2015), and Chinco and Mayer (2015).}\)
potential resident belief $\mu_{r}^{agg}(N_{0},x,z)$ to be the unique solution to

$$1 = N_{0} \int_{\Theta} D \left( k(1 + e^{\mu_{agg}r(N_{0})x/\epsilon})N_{0}^{1/\epsilon} - p_{h}^b(e^{\mu(\theta)x}N_{0}) \right) f_{r}(\theta) d\theta,$$

where existence and uniqueness are established in the proof of Lemma 3. This aggregator takes the formula for $p_{h}^b(N_{0},x,0)$ from Proposition 1 and calculates the value that must replace $\overline{\mu}$ such that the resulting price clears the market in equilibria when $z > 0$ and no developers hold land. In this sense, $\mu_{r}^{agg}(N_{0},x,z)$ describes how the market aggregates the disparate beliefs of potential residents. In particular, $\mu_{r}^{agg}(N_{0},x,z)$ always lies below the maximal potential resident belief $\mu_{r}^{max}$, as shown in the proof of Lemma 3.

The following lemma describes the effect of disagreement on the aggregate potential resident belief under certain conditions:

Lemma 3. If $\int_{\Theta} \theta f_{r}(\theta) d\theta = 0$ and $\text{supp} f_{r} \subset [-\overline{\mu}/z, \overline{\theta}_{d}^{max}]$, then $\mu_{r}^{agg}(N_{0},x,z) > \overline{\mu}$ and $\mu_{r}^{agg}(N_{0},x,z) = \overline{\mu} + o(z)$ as $z \to 0$.

When the mean potential resident belief is $\overline{\mu}$ and these beliefs fall within certain bounds, the belief aggregator exceeds the mean belief $\overline{\mu}$. The Miller (1977) effect is in force: with disagreement, the aggregate belief implied by market-clearing is greater. However, for small values of $z$ the Miller (1977) effect is only second order. To the first order, the decline in demand from pessimists perfectly offsets the increase in demand from optimists, so small disagreement does not change the aggregate belief.

The following proposition characterizes the equilibrium house price under disagreement for all values of $N_{0}$:

Proposition 3. The equilibrium house price at $t = 0$ equals

$$p_{h}^b(N_{0},x,z) = \begin{cases} 2k & \text{if } N_{0} \leq e^{-\mu_{d}^{max}x} \text{ (unconstrained)} \\ k + ke^{\mu_{d}^{max}x/\epsilon}N_{0}^{1/\epsilon} & \text{if } e^{-\mu_{d}^{max}x} < N_{0} < N_{0}^{*}(x,z) \text{ (intermediate)} \\ k(1 + e^{\mu_{agg}r(N_{0},x,z)x/\epsilon})N_{0}^{1/\epsilon} & \text{if } N_{0} \geq N_{0}^{*}(x,z) \text{ (constrained)}. \end{cases}$$

Proposition 3 shows how the allocation of land and housing among developers and potential residents affects the price of housing at $t = 0$. There are two important differences between cities with different initial demand levels in terms of how prices aggregate beliefs. First, developer beliefs matter more in intermediate cities, whereas potential resident beliefs

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13The condition on $\text{supp} f_{r}$ implies that $0 \leq \mu(\theta) \leq \mu_{d}^{max}$ for all potential residents, which leads the demand curve for housing to be globally convex with respect to $\theta$. Global convexity implies that disagreement (holding the price constant) stimulates the demand of optimists more than it attenuates the demand of pessimists. Thus, Jensen’s inequality implies that the price that clears the market under disagreement exceeds the market-clearing price with agreement, meaning that $\mu_{r}^{agg}(N_{0},x,z) > \overline{\mu}$. 

13
matter more in constrained cities. Given the stark assumptions of our model (in particular, the linear production technology for houses as well as the absence of an alternate use for land), prices in intermediate cities reflect only the beliefs of developers. Recent research has measured owner-occupant beliefs about the future evolution of house prices. Data on the expectations of homebuilders would supplement the research on owner-occupant beliefs to explain house price movements, especially in cities at intermediate levels of development.

The second difference between intermediate-demand and high-demand cities is how they aggregate the beliefs of the relevant class of agents. In intermediate cities, prices reflect the most optimistic belief $\mu_d^{\text{max}}$. Other than the maximal value of its support, all other information encoded in the distribution $f_d$ of $\theta$ across developers is irrelevant for prices. In contrast, the entire distribution $f_r$ of potential resident beliefs matters for house prices when $N_0 \geq N^*_0(x, z)$. This stark contrast depends on the absence of other constraints on developer size such as risk aversion or capital constraints. The more general point is that when land is held by potential residents, prices need not reflect the most optimistic belief. This is because residents derive utility from housing that may not be correlated with expected capital gains. Moreover, because these utility benefits exhibit diminishing returns, homeownership will tend to be more dispersed than land ownership.

Prices reflect more optimistic beliefs in intermediate cities than in constrained cities when $\mu_d^{\text{max}} > \mu_r^{\text{agg}}(N_0, x, z)$ for all $N_0 \geq N^*_0(x, z)$. The following assumption on the belief distributions is necessary for this relationship to hold:

**Assumption 5.** $e^{\mu_d^{\text{max}}x/\epsilon} - e^{\mu_r^{\text{max}}x/\epsilon} < 1$ and $\int_\Theta \left( 1 + e^{\mu_d^{\text{max}}x/\epsilon} - e^{\mu_r(\theta)x/\epsilon} \right)^{-\epsilon} f_r(\theta) d\theta < 1$.

Assumption 5 holds when $f_r = f_d$, so that the distribution of beliefs is the same for each class of agents, but it may hold even if some potential residents are more optimistic than the most optimistic developer. The assumption fails if there is a sufficiently large group of residents with very optimistic beliefs. We invoke Assumption 5 for the purpose of analyzing the effect of disagreement on the house price boom at $t = 0$.

### 3.3 The Effect of Disagreement on House Prices

We turn now to the model’s key result, which concerns the effect of disagreement on the price of housing at $t = 0$:

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Proposition 4. The effect of disagreement on the house price at \( t = 0 \) is given by

\[
p_h^0(N_0, x, z) - 1 = \begin{cases} 
0 & \text{if } N_0 \leq e^{-\mu_d^{\text{max}x}} \\
\frac{1}{2} (e^{\mu_d^{\text{max}x}/\epsilon} N_0^{1/\epsilon} - 1) & \text{if } e^{-\mu_d^{\text{max}x}} < N_0 \leq e^{-\pi x} \\
(e^{\mu_d^{\text{max}x}/\epsilon} - e^{\pi x/\epsilon}) N_0^{1/\epsilon} & \text{if } e^{-\pi x} < N_0 \leq 1 \\
(1 + e^{\pi x/\epsilon}) N_0^{1/\epsilon} - (N_0^{1/\epsilon} - 1) & \text{if } 1 < N_0 < N_0^*(x, z) \\
e^{\mu_d^{agg}(N_0, x, z)/\epsilon} - e^{\pi x/\epsilon} & \text{if } N_0 \geq N_0^*(x, z).
\end{cases}
\]

The increase is positive for \( N_0 \in (e^{-\mu_d^{\text{max}x}}, 1] \) and is strictly maximized at \( N_0 = 1 \). If \( \text{supp } f_r \subset [-\pi/\epsilon, \theta_d^{\text{max}x}] \) and \( \int_\Theta \theta f_r(\theta) d\theta = 0 \), then the increase is also positive for all \( N_0 > 1 \), and the marginal effect of small disagreement on the price of housing,

\[
\frac{\partial p_h^0(N_0, x, 0)}{\partial z} \left/ \frac{p_h^0(N_0, x, 0)}{p_h^0(N_0, x, 0)} \right. = \begin{cases} 
0 & \text{if } N_0 < e^{-\pi x} \\
e^{\mu_d^{\text{max}x} - \pi x/\epsilon} N_0^{1/\epsilon} & \text{if } e^{-\pi x} \leq N_0 \leq 1 \\
1 & \text{if } N_0 > 1,
\end{cases}
\]

is positive only for \( e^{-\pi x} \leq N_0 \leq 1 \).

The first part of Proposition 4 calculates the relative effect of disagreement on \( p_h^0(N_0, x, z) \) by comparing the price formulas in Propositions 1 and 2. We now describe the effect of disagreement on the price of housing in each regime using the expressions in Proposition 4.

When \( N_0 \leq e^{-\mu_d^{\text{max}x}} \), all developers agree that \( p_1^h = k \) because the city will remain unconstrained at \( t = 1 \). Disagreement between developers on how to interpret the shock \( x \) is irrelevant for today’s price. Consistent with basic intuition, a fully unconstrained city reacts similarly to an expected shock under agreement and disagreement.

When \( e^{-\mu_d^{\text{max}x}} < N_0 \leq e^{-\pi x} \), the most optimistic developers expect the city to be constrained so that \( p_1^h > k \), but a developer with the average belief does not. As a result, disagreement raises the price of housing. This increase is larger when \( N_0 \) is greater because the price the most optimistic developer expects at \( t = 1 \) rises with the level of demand.

The analysis of the \( e^{-\pi x} < N_0 \leq 1 \) case is similar, except that now the average developer believes the city will be constrained in the future. Within this range, \( p_1^h > k \) under both the average and most optimistic developer belief. The effect of disagreement reflects the extent to which the optimistic developer belief of \( p_1^h \) exceeds the average belief, with this difference appearing in the numerator. As \( N_0 \) increases, \( p_h^0(N_0, x, 0) \) places more weight on
beliefs about \( t = 1 \) relative to the user cost at \( t = 0 \), so the effect of disagreement on this price increases as well.

The effect of disagreement is most subtle when \( 1 < N_0 < N_0^*(x, z) \). In this range, disagreement changes the equilibrium from one in which potential residents own all space to one in which the most optimistic developers hold some land. This change alters the house price in two opposing ways, corresponding to the two terms in the numerator in Proposition 4. The first term is positive, as beliefs about \( p_{h_1}^0 \) rise from \( ke^{\bar{p}_x} N_0^{1/\epsilon} \) to \( ke^{\mu_{\max}_d} N_0^{1/\epsilon} \). The second term is negative, as the flow valuation of the marginal buyer at \( t = 0 \) falls from \( kN_0^{1/\epsilon} \) to \( k \). These terms reflect a change in land ownership at the margin: under agreement, the marginal buyer is a potential resident whose housing utility equals \( kN_0^{1/\epsilon} \); under disagreement, the marginal buyer is a developer whose flow value of housing is \( k \). The net price effect strictly decreases with \( N_0 \) because utility crowd-out increases relative to \( N_0^{1/\epsilon} \) as \( N_0 \) gets larger.

When \( N_0^*(x, z) < \infty \), the effect of disagreement eventually reaches the level given in the final regime of Proposition 4. The effect is positive when \( \mu_{agg}^r(N_0, x, z) > \bar{\mu} \) for all \( N_0 \geq N_0^*(x, z) \), which holds given the conditions in the proposition as shown by Lemma 3. When \( N_0^*(x, z) = \infty \), for large \( N_0 \) the effect of disagreement on the house price asymptotes to \( (e^{\mu_{\max}_d x/\epsilon} - e^{p_x/\epsilon} - 1)/(1 + e^{p_x/\epsilon}) \), which the proof of Proposition 4 shows is positive.

The key result is that the effect of disagreement in the \( N_0 \geq N_0^*(x, z) \) regime is less than that at \( N_0 = 1 \). This comparison depends on Assumption 5, which guarantees that \( \mu_{agg}^r(N_0, x, z) < \mu_{\max}^d \), and on Assumption 2, which leads to dispersion of homeownership among potential residents of all beliefs.

The last part of Proposition 4 isolates the effect of disagreement by studying a small increase in \( z \) from \( z = 0 \). As shown by Lemma 3, this increase has no first-order effect on \( \mu_{agg}^r(N_0, x, z) \) (under the given conditions on \( f_r \)). In contrast, the increase in \( z \) does raise the maximal developer belief \( \mu_{\max}^d \) to the first order because \( \partial \mu_{\max}^d / \partial z = \theta_{\max}^d > 0 \). As a result, the marginal effect of small disagreement is positive only in the intermediate region of \( N_0 \).

In summary, disagreement raises the price of housing everywhere except cities where the level of demand is very low and possibly cities where the level of demand is very high but many extreme optimists and pessimists exist. For specifications of the joint distribution of resident and developer beliefs that satisfy Assumption 5 (such as identical distributions in the two subpopulations), disagreement raises the price most in cities at an intermediate level of development.

### 3.4 The Variation in Price Booms across Cities

Proposition 5 characterizes how the house price boom under disagreement varies across cities.
Proposition 5. The house price boom under disagreement,

\[
p_h^0(N_0, x, z) = \begin{cases} 
0 & \text{if } N_0 \leq e^{-\mu_{d}^{\max}} \\
\frac{1}{2}(e^{\mu_{d}^{\max}/\epsilon N_0^{1/\epsilon} - 1}) & \text{if } e^{-\mu_{d}^{\max}} < N_0 \leq 1 \\
\frac{1}{2}(e^{\mu_{d}^{\max}/\epsilon - 2 + N_0^{-1/\epsilon}}) & \text{if } 1 < N_0 < N_0^*(x, z) \\
\frac{1}{2}(e^{\mu_{agg}(N_0, x, z)/\epsilon - 1}) & \text{if } N_0 \geq N_0^*(x, z),
\end{cases}
\]

is strictly maximized at \( N_0 = 1 \).

As in Proposition 1, we define the price boom as the effect of the shock \( x \) on \( p_h^0(N_0, 0, z) \). This boom can be decomposed into the product of \( p_h^0(N_0, x, 0) / p_h^0(N_0, 0, 0) \) (the marginal effect of \( x \) when \( z = 0 \) given by Proposition 1) and \( p_h^0(N_0, x, z) / p_h^0(N_0, x, 0) \) (the marginal effect of \( z \) given by Proposition 4). The former monotonically increases in the level of initial demand \( N_0 \), whereas the latter strictly peaks at \( N_0 = 1 \). Proposition 5 shows that the combined effect also strictly peaks at \( N_0 = 1 \), meaning that with disagreement the result that demand shocks raise prices the most in constrained cities no longer holds.

The intuition behind Proposition 5 is similar to that of Proposition 4. Cities with low initial demand experience no price boom because all developers agree that they will remain unconstrained at \( t = 0 \). For intermediate cities where \( e^{-\mu_{d}^{\max}} < N_0 \leq 1 \), prices rise according to the beliefs of the most optimistic developer. For intermediate cities with \( 1 < N_0 < N_0^*(x, z) \), prices rise less when \( N_0 > 1 \) because of the utility crowd-out of homeowners by developers. Finally, prices rise for \( N_0 \geq N_0^*(x, z) \) according to the aggregate beliefs of all potential residents. Under Assumption 5, this aggregate belief falls short of the most optimistic developer belief, so the price boom is largest when \( N_0 = 1 \).

To illustrate the variation in the price boom across cities, Figure 2 plots the expression from Proposition 5 across different values of \( N_0 \), both for a positive value of \( z \) and for \( z = 0 \). We set \( f_r = f_d \) so the conditions of Assumption 5 hold. As can be seen in the figure, disagreement amplifies the boom everywhere except in cities with small initial demand where disagreement has no effect. The amplification is largest in cities with intermediate values of initial demand, leading the boom to be largest in the case of disagreement at \( N_0 = 1 \). The boom in the case of agreement rises monotonically with respect to the level of initial demand.

3.5 Disagreement and Welfare

Disagreement can reallocate space from pessimists to optimists. This reallocation destroys welfare if some of the pessimists are potential residents with high flow utility and some of the

\[\text{Here we are using the fact that } p_h^0(N_0, 0, z) = p_h^0(N_0, 0, 0) \text{ because } z \text{ becomes irrelevant when } x = 0.\]
optimists are developers or potential residents with lower flow utility. To formally analyze the effect of disagreement on welfare, we adopt the “belief-neutral Pareto efficiency” criterion proposed by Brunnermeier, Simsek and Xiong (2014) as a welfare measure for models with heterogeneous beliefs. An allocation is belief-neutral Pareto efficient if it is Pareto efficient under all linear combinations of agent beliefs. Proposition 6 shows that disagreement reduces welfare in intermediate and constrained cities.

**Proposition 6.** The equilibrium allocation is belief-neutral Pareto efficient if

\[ z = 0 \] or \[ N_0 \leq e^{-\mu_{\text{max}} x} \] and is belief-neutral Pareto inefficient otherwise.

While disagreement may lead only to welfare-neutral transfers in the stock market, this result demonstrates that disagreement tends to lower welfare in the housing market.

The reallocations that improve welfare when \( z > 0 \) are as follows. When \( e^{-\mu_{\text{max}} x} < N_0 < N^*_0(x, z) \), there exists a potential resident who chooses not to buy despite having flow utility \( v > k \). The resource cost of building a house at \( t = 0 \) instead of \( t = 1 \) is \( k \), so there exists a cash transfer from this potential resident to a developer that makes them both better off if the developer builds a house and gives it to the potential resident. For \( N_0 \geq N^*_0(x, z) \), there exist potential residents with flow utilities \( v^1 \) and \( v^2 \) such that \( v^1 < v^2 \) and the potential resident with \( v = v^1 \) buys whereas the one with \( v = v^2 \) does not. With a suitable cash transfer, changing which potential resident owns the house improves the welfare of both. Under the \( z = 0 \) equilibrium allocation, these situations never occur.

### 4 Extensions and Additional Predictions

**Equity Financing.** The developers in the baseline model raise any needed funds at \( t = 0 \) using debt. Appendix B presents an extension in which developers may raise funds only through equity offerings. The analysis formalizes results that we explore empirically in Section 5, in which we examine the market value and short-selling of the equity of public developers during the 2000-2006 US housing boom.

In this extension, *equity investors* constitute a third class of agents. Across equity investors, the distribution of beliefs \( f_i \) about \( \mu_{\text{true}} \) satisfies Assumption 3, which guarantees the existence of a most optimistic equity investor, and Assumption 4, which ensures disagreement when \( z > 0 \). Some of the developers endowed with land at \( t = 0 \) may raise funds by selling claims on their \( t = 1 \) liquidation values to these investors.\(^{16}\) Each equity investor may borrow freely at a rate of 0 to finance positive purchases of these claims. In contrast, to sell these claims short, equity investors must pay a positive proportional fee. Furthermore,\(^{16}\)The developers who cannot access the equity market represent small firms and nonprofit landowning entities like governments and Native American tribes that are not able to issue equity.
each equity investor may sell short a limited number of claims. In equilibrium, a price exists for the claim on each developer able to access the equity market such that the value of the claims sold by the developer equals the net quantity demanded by equity investors.

The following proposition characterizes the price of housing, the allocation of land holdings, the price of developer equity, and the total short position by equity investors at \( t = 0 \). We define \( p_h^0(N_0, x, z, f_r, f_d) \) to be the equilibrium value of \( p_h^0 \) in Proposition 2 given the potential resident belief distribution \( f_r \) and the developer belief distribution \( f_d \), and we denote \( \theta_i^{\text{max}} \) and \( \mu_i^{\text{max}} = \mu(\theta_i^{\text{max}}) \) to be the type and belief of the most optimistic equity investor.

**Proposition 7.** If \( xz = 0 \), then the aggregate value of short claims equals zero, and there exists an equilibrium in which no equity issuance nor land purchases occur. If \( xz > 0 \):

- if \( \sum_{\theta > \theta_i^{\text{max}}} L_0 = 0 \) then the equilibrium house price equals \( p_h^0(N_0, x, z, f_r, f_i) \);
- if \( \sum_{\theta < \theta_i^{\text{max}}} L_0/S > e^{-\mu_i^{\text{max}}x} \) then there exists \( N_0 \) for which the following all hold:
  
  (a) some developers issue equity with positive value,
  
  (b) \( \sum (L_0^{\text{buy}})^* > \sum (H_0^{\text{bld}})^* \) across developers issuing equity,
  
  (c) the total short position in this equity is positive for some values of the short fee,
  
  (d) the equity price for each such developer exceeds the price under \( x = 0 \), and
  
  (e) the equity price for each such developer falls from \( t = 0 \) to \( 1 \) iff \( \mu_i^{\text{max}} > \mu_{\text{true}} \).

If none of the developers endowed with land are more optimistic than the most optimistic equity investor (for example, if the developer and investor belief distributions coincide), the pricing formula in Proposition 2 carries over to the model with equity financing with one difference: now the most optimistic equity investor belief replaces the most optimistic developer belief. The non-monotonicity of the house price boom and disagreement price effect also carry over as long as \( \theta \leq \theta_i^{\text{max}} \) for non-landowning developers.\(^{18}\) The most optimistic investor prices all developer equity because long positions are unlimited while short positions are costly. When \( \sum_{\theta > \theta_i^{\text{max}}} L_0 = 0 \), all developers are willing to sell equity backed by their landholdings to investors or to sell their landholdings at the optimistic investor valuation. As

\( ^{17} \)In this extension, land that remains undeveloped at the end of \( t = 0 \) pays a small positive dividend at the beginning of \( t = 1 \). Proposition 7 reports the limiting equilibrium price as this dividend goes to 0. The sole purpose of this dividend is to ensure the existence of equilibrium when \( p_h^0(N_0, x, z) = 2k \).

\( ^{18} \)In this case \( \theta_i^{\text{max}} \leq \theta_i^{\text{max}} \), so Assumption 5 (on which Propositions 4 and 5 rely) holds with \( \mu_i^{\text{max}} \) in place of \( \mu_d^{\text{max}} \).
a result, the equilibrium land price coincides with the optimistic investor valuation, leading to an equilibrium house price of $p_h^0(N_0, x, z, f_r, f_i)$.\(^{19}\)

Proposition 7 also characterizes quantities in the equity and land markets without and with disagreement. Without disagreement, short-selling never occurs because all investors agree on the equity valuations and the short fee is positive. Equity issuance and land purchases by developers are not guaranteed to occur because developers are indifferent between selling land, selling equity, and holding land until $t = 1$.

With disagreement, equity issuance, land purchases by equity-issuing developers, and short-selling of that equity all occur in equilibrium as long as the short fee is small enough and pessimistic developers without equity market access hold enough land. With enough pessimistic developers, other developers must raise funds from optimistic investors to buy out the pessimists and satisfy the expected demand of potential residents. Equity-issuing developers buy land in excess of their immediate construction needs, resembling the optimistic developers characterized by Proposition 2.

**Housing Rental Market.** In the baseline model, potential residents derive housing utility only from owning and may own only one house. To explore the importance of these restrictions, Appendix C presents an extension in which rental contracts are available and potential residents may operate as landlords.

In this extension, potential residents may buy any positive amount of housing. They choose how much housing to lease as landlords and how much to keep as owner-occupied housing. Potential residents may also choose to rent housing as tenants. A fraction $\chi$ of potential residents receive housing utility $v$ if and only if they occupy at least one unit of housing as a tenant, and the remaining potential residents receive $v$ if and only if they occupy at least one unit of housing as an owner-occupant. Rental prices at $t = 0$ and $t = 1$ clear the market, so that the quantity of housing chosen to be leased by landlords equals the quantity chosen to be rented by tenants.

We define $N_0^\ast(x, z, f_r)$ to be the value of $N_0^\ast$ in Proposition 2 given the potential resident belief distribution $f_r$, and we define the distribution $f_r^\chi$ by $f_r^\chi(\theta) = \chi 1_{\theta_{\max}} + (1 - \chi) f_r(\theta)$. The following proposition describes the $t = 0$ equilibrium in the rental extension:

**Proposition 8.** If $xz = 0$, $\chi$ equals the share of the housing stock that is rented. If $xz > 0$:

- the equilibrium house price equals $p_h^0(N_0, x, z, f_r^\chi, f_i)$;

\(^{19}\)Appendix B analyzes in detail the case in which $\sum_{\theta > \theta_{\max}} L_0 > 0$. In this case, some landowning developers are more optimistic than the most optimistic investor. There may exist values of $N_0$ at which these optimistic developers hold land in equilibrium, with the $t = 0$ house price independent of the beliefs and endowments of all other developers.
there exists $\chi^*(x,z) \in (0,1]$ such that the house price boom is strictly maximized at $N_0 = 1$ if $\mu_r^{\max} < \mu_d^{\max}$ or if $\mu_r^{\max} \geq \mu_d^{\max}$ and $\chi < \chi^*(x,z)$; and

the house price boom depends on $\chi$ only if $N_0 \geq N^*_0(x,z,f^\chi)$, in which case it increases.

As shown by Proposition 8, the equilibrium house price assigns additional weight to the optimistic resident belief in proportion to the underlying share of potential residents who prefer renting. This skewing occurs because only the most optimistic potential residents become landlords in equilibrium.

Proposition 8 further shows that the non-monotonicity of the house price boom characterized by Proposition 5 carries over to this setting under certain conditions. If $\mu_r^{\max} < \mu_d^{\max}$, then the house price boom remains maximized at $N_0 = 1$ for all $\chi$ because the developers pricing housing at that point are more optimistic than the residents pricing housing in markets where developers do not participate. In the more interesting case when $\mu_r^{\max} \geq \mu_d^{\max}$, the price boom is maximized at $N_0 = 1$ only for sufficiently small values of $\chi$. This $\chi < \chi^*(x,z)$ constraint need not be very restrictive—if $\mu_r^{\max} = \mu_d^{\max}$, then $\chi^*(x,z) = 1$, so the boom remains non-monotonic as long as some positive measure of potential residents prefer owner-occupancy to renting.

The last part of Proposition 8 delivers the empirical prediction that among identical cities where no land remains, cities in which a higher share of housing is rented without disagreement experience larger house price booms with disagreement. We explore this prediction in Section 5.

Continuous Housing Supply Elasticity. A key statistic used to analyze house price booms is the elasticity of housing supply (Glaeser, Gyourko and Saiz, 2008; Mian and Sufi, 2009). In the baseline model, this elasticity is zero when all land is used for housing and infinite when some undeveloped land remains. Disagreement amplifies the house price boom most when $e^{-\mu_d^{\max}x} < N_0 < N^*_0(x,z)$, the parameter region in which supply is perfectly elastic at $t = 0$ but is expected by the optimistic developers to be perfectly inelastic at $t = 1$. This characterization suggests that disagreement amplifies price booms most when supply is elastic today but expected to be inelastic tomorrow. To see how robust this conclusion is, Appendix D extends the rental market model to the case in which the supply elasticity declines continuously with the level of initial demand.

In this extension, developers may rent out undeveloped land on spot markets each period to firms that use the city’s land as an input. The land demand from these firms at $t = 0$ and $t = 1$ is given by a continuously differentiable, decreasing, positive function of the spot rental rate. The limiting values of this function for small and large rents are sufficiently extreme that a unique equilibrium exists, and this function becomes weakly more inelastic for larger rents.
Given a house price $p^h_t$, a unique partial equilibrium exists in which developers optimize and the land markets clear. We define the elasticity of supply to be the partial derivative of the log of the housing stock chosen by developers with respect to $p^h_t$, all normalized by the house rent $r^h_t$. As shown in Appendix D, there exists a continuous, decreasing function $\epsilon^s : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that the supply elasticity when $z = 0$ equals $\epsilon^s(N_0)$.

It is difficult to provide an exact solution for the house price boom (a counterpart for Proposition 5) in this setting. Instead, the following proposition solves for the marginal impact of the shock $x$ on the equilibrium house price $p^h_0(N_0, x, z, \chi)$ when $x$ or $z$ is zero. We then use the formulas from these special cases to describe the distinct roles played by the current and future housing supply elasticity.

**Proposition 9.** The following approximation holds exactly for $x = 0$ or $z = 0$:

$$
\frac{\partial \log p^h_0(N_0, x, z, \chi)}{\partial x} \approx \left( \frac{\epsilon^s(N_0) \theta^\max d + \chi \epsilon r^\max r + (1 - \chi) \epsilon \theta^avg r}{\epsilon^s(N_0) + \epsilon} \right) \frac{(\epsilon^s(e^{\mu x} N_0) + \epsilon)^{-1}}{1 + e^{-\int_0^\theta \epsilon^s(e^{\mu x} N_0) + \epsilon)^{-1} dx}}
$$

where $\theta^avg r = \int \theta f_r(\theta) d\theta$.

Proposition 9 confirms the intuition from the main model on the distinct roles played by the supply elasticities at $t = 0$ and $t = 1$. The fraction multiplying $z$ aggregates beliefs across market participants. When the current elasticity $\epsilon^s(N_0)$ is higher, more weight is placed on the optimistic developer type $\theta^\max d$ because these developers constitute a larger share of the marginal buyers in the market. A larger $\epsilon^s(N_0)$ implies a greater influence of disagreement on the house price today. Similarly, a higher $\chi$ increases the share of marginal buyers who are landlords and raises the weight on the optimistic resident type $\theta^\max r$.

In contrast, a greater value of $\epsilon^s(e^{\mu x} N_0)$—the supply elasticity at $t = 1$ under the mean belief—lowers the house price boom at $t = 0$. A larger supply elasticity at $t = 1$ implies smaller pass-through of the shock $x$ to the price at $t = 1$. This pass-through is given by $(\epsilon^s(e^{\mu x} N_0) + \epsilon)^{-1}$. Similarly, a greater value of $\epsilon^s(e^{\mu x} N_0)$ lowers the value of the integral, which captures the expected price tomorrow relative to today. When this ratio is lower, the

---

20 Normalizing by $r^h_t$ instead of $p^h_t$ allows us to use the same function to analyze the supply elasticity at $t = 0$ and $t = 1$. Given the finite horizon of the model, the price at $t = 0$ is roughly double the price at $t = 1$, so supply would be about twice as elastic at $t = 0$ if we normalized by $p^h_t$. A unique equilibrium rent exists for $\chi > 0$; when $\chi = 0$ we choose the rent that obtains as the limit as $\chi \rightarrow 0$, which is a valid equilibrium rent for $\chi = 0$.

21 The beliefs of non-landlord potential residents appear through only $\theta^avg r$, and hence their disagreements relative to this average are irrelevant for this formula. The intuition for this result is similar to the intuition for the limit result in Lemma 3. When $x$ is near 0, the variation in beliefs about $p^h_t$ among potential residents is small for any $z$. This small disagreement does not alter potential resident housing demand because the decline in demand from pessimists perfectly offsets the increase in demand from optimists.
pass-through of the shock \( x \) to the price at \( t = 0 \) is lower.

In summary, the price boom is largest when \( \epsilon^s(N_0) \) is large but \( \epsilon^s(e^{\pi x} N_0) \) is small. This combination occurs when the shock \( x \) pushes the city from having a high supply elasticity in the present to possibly having a low supply elasticity in the future. The possibility of such a transition amplifies the role of disagreement relative to cases in which transition to low supply elasticity is unlikely or has already occurred. Thus the intuition from the baseline model that disagreement affects the size of the house price boom most for cities at an intermediate level of development translates to this more general setting.

### 5 Stylized Facts of the US Housing Boom and Bust

This section uses data from the US housing boom between 2000 and 2006 to provide evidence consistent with the model’s predictions about the house price boom at \( t = 0 \). We first document the importance of the relationship between the price of raw land and the price of housing across cities, which supports the model’s focus on how disagreement interacts with potential land constraints. We then describe the speculative behavior in land markets among public homebuilders, who resemble the optimistic developers in the model extension in which developers issue equity. Last, we show how the model can be used to understand the booms in the cross-section of US cities, as well as across neighborhoods within cities.

#### 5.1 The Central Importance of Land Prices

A key assumption of the model is that housing supply is limited in the long run by development constraints. These constraints lead land prices to rise during a housing boom, as developers anticipate the exhaustion of land. As a result, house prices and land prices rise in unison as shown by the result in Lemma 2 that \( p_h^0 = p_l^0 + 2k \) in equilibrium.

Tracing house price increases to land prices distinguishes our model from “time-to-build.” Traditionally, housing supply has been modeled as inelastic in the short run and elastic in the long run (DiPasquale and Wheaton, 1994; Mayer and Somerville, 2000). This paradigm described the US housing market very well for a time. Topel and Rosen (1988) show that essentially all variation in house prices between 1963 and 1983 in the US came from changes to the construction cost of structures. Temporary shortages of inputs needed to build a house, such as drywall and skilled labor, could explain this pattern, with the fluctuations in these input prices causing house price cycles.

Between 1983 and 2000, a secular shift occurred in housing supply in the US. Land prices became a much larger share of house prices (Davis and Heathcote, 2007), especially in certain cities (Davis and Palumbo, 2008). A large literature, surveyed by Gyourko (2009), has attributed this change to the rise of government regulations restricting housing supply.
These rules bound city growth by limiting the number of building permits that are issued to developers. When demand to live in the city rises, land prices increase because the city cannot expand.

Developers in a city without supply restrictions today might expect them to arrive in the future. Anticipating these regulatory changes, developers bid up land prices immediately after a demand shock, even in the absence of current building restrictions. In such a city, supply is elastic in the short run, but inelastic in the long run—it’s “arrested development.” In the baseline model, cities at an intermediate development level exhibit an extreme case of arrested development, with an infinite supply elasticity at $t = 0$ and a zero supply elasticity at $t = 1$ if all land is developed. The equilibrium presented in Proposition 9 considers a more general case in which the supply elasticity declines continuously with the level of initial demand.

Under arrested development, a nationwide housing demand shock can increase land prices everywhere, not just in cities where regulations currently restrict supply. Land prices rise even in areas with rapid construction. In contrast, time-to-build predicts construction cost increases and not land price increases. If temporary input shortages are driving house prices, then land prices, which are fully forward looking, should remain flat.

To assess the relative importance of land prices, we gather data on land prices and construction costs at the city level between 2000 and 2006. We use land prices measured directly from parcel transactions during this time. This approach contrasts with that used by Davis and Heathcote (2007) and Davis and Palumbo (2008), who measure land prices as the residual when construction costs are subtracted from house prices. A direct measure of land prices addresses concerns that such residuals capture something other than land prices between 2000 and 2006. The land price data we use are the indices constructed by Nichols, Oliner and Mulhall (2013). Using land transaction data, they regress prices on parcel characteristics and then derive city-level indices from the coefficients on city-specific time dummies.

We measure construction costs using the R.S. Means construction cost survey. This survey asks homebuilders in each city to report the marginal cost of building a square foot of housing, including all labor and materials costs. Survey responses reflect real differences across cities. In 2000, the lowest cost is $54 per square foot and the highest is $95; the mean is $67 per square foot and the standard deviation is $9. This survey has been used to study the time series and spatial variation in residential construction costs (Glaeser and Gyourko, 2005; Gyourko and Saiz, 2006; Gyourko, 2009).

As shown by Lemmas 1 and 2, the assumptions of our model imply that house prices must equal land prices plus construction costs: $p^h_t = p^l_t + k_t$. Log-differencing this equation
between 2000 and 2006 yields

\[ \Delta \log p^h = \alpha \Delta \log p^l + (1 - \alpha) \Delta \log k, \]

where \( \Delta \) denotes the difference between 2000 and 2006 and \( \alpha \) is land’s share of house prices in 2000. The factor that matters more should vary more closely with house prices across cities. Because \( \alpha \) and \( 1 - \alpha \) are less than 1, the critical factor should also rise more than house prices do.

Figure 3 plots for each city the real growth in construction costs and land prices between 2000 and 2006 against the corresponding growth in house prices. Construction costs did rise during this period, but they rose substantially less than land prices, and construction cost increases display very little variation across cities. The time-to-build hypothesis, then, does explain some of the level of house price increases in the US during the boom, but none of the cross-sectional variation. Land prices display the opposite pattern, rising substantially and exhibiting a high correlation with house prices. Each city’s land price increase also exceeds its house price increase. This evidence underscores the central importance of land prices for understanding the cross-section of the house price boom, and broadly supports the relative contribution of arrested development over time-to-build.

5.2 Supply-Side Speculation by Homebuilders

Proposition 2 predicts that as long as they are not endowed with too much land, optimistic developers amass land beyond their immediate construction needs at \( t = 0 \) in intermediate cities. Proposition 7 extends this result to the case in which developers finance themselves with equity and offers additional predictions about the developer equity market. We examine these predictions among a class of developers for whom rich data are publicly available: public homebuilders. We focus on the eight largest firms and hand-collect landholding data from their annual financial statements between 2001 and 2010.\(^22\)

The eight equity-financed large firms we study nearly tripled their landholdings between 2001 and 2005, as shown in Figure 4(a). Consistent with Propositions 2 and 7(a) and 7(b), these land acquisitions far exceed land needed for new construction. Annual home sales increased by 120,000 between 2001 and 2005, while landholdings increased by 1,100,000 lots. One lot can produce one house, so landholdings rose more than nine times relative to home sales. In 2005, Pulte changed the description of its business in its 10-K to say, “We consider land acquisition one of our core competencies.” This language appeared until 2008, when it was replaced by, “Homebuilding operations represent our core business.”

\(^{22}\)Our analysis complements Haughwout et al. (2012), an empirical study of the homebuilding industry that presents similar facts from different data.
Having amassed large land portfolios, these firms subsequently suffered significant capital losses. Figure 4(b) documents the dramatic rise and fall in the total market equity of these homebuilders between 2001 and 2010. Homebuilder stocks rose 430% and then fell 74% over this period. By Proposition 7(d), the rise is consistent with a positive shock $x > 0$ at $t = 0$. If we interpret the period between 2006 and 2010 as that between $t = 0$ and $t = 1$, then by Proposition 7(e) the losses are consistent with a realization of $\mu^{true}$ below the optimistic investor belief $\mu_{i}^{max}$.

The majority of the losses borne by homebuilders arose from losses on the land portfolios they accumulated from 2001 to 2005. In 2006, these firms began reporting write-downs to their land portfolios. At $29$ billion, the value of the land losses between 2006 and 2010 accounts for 73% of the market equity losses over this time period. The homebuilders bore the entirety of their land portfolio losses. The absence of a hedge against downside risk supports the theory that homebuilder land acquisitions represented optimistic beliefs.

It is hard to argue that this rise and fall of equity prices reflects any monopoly rents homebuilders earned by building houses during this period. During the boom, homebuilding was extremely competitive. Haughwout et al. (2012) document that the largest ten homebuilders had less than a 30% market share throughout the boom, with firms outside the largest sixty constituting over half of market share. Although some consolidation occurred between 2000 and 2006, these numbers portray an extremely competitive market. If anything, consolidation may reflect purchases by optimistic firms of pessimists who chose to abstain from land speculation.

Consistent with Prediction 7(c), these homebuilders witnessed heightened short-selling of their equity during the boom. Figure 4(c) plots the distribution of the average monthly short interest ratio, defined as the ratio of shares currently sold short to total shares outstanding, across all industries between 2000 and 2006. The short interest of homebuilder stocks lies in the 95th percentile, meaning that investors short-sold this industry more than nearly all others during the boom. As a point of comparison, the short interest in homebuilders was triple that in investment banks, another industry exposed to housing at this time. The short interest in homebuilders provides direct evidence of disagreement over the value of their land portfolios.

Several recent papers argue that optimism about house prices was widespread between 2000 and 2006. For instance, Foote, Gerardi and Willen (2012) document twelve facts about the mortgage market during this time inconsistent with incentive problems between

\[23\text{An earlier draft of this paper provided the time series of short interest in homebuilder stock from 2001 to 2010. Short interest rose from 2001 to 2006, but rose even further from 2006 to 2009. Homebuilder short interest was highest as the bust was beginning. This peak may indicate that disagreement reached its peak after the boom, complicating the idea that disagreement was high during the boom. Alternatively, the late peak could indicate that shorting is more attractive for pessimists when they anticipate a bust in the near future.}\]
borrowers and lenders, but consistent with beliefs of borrowers and lenders that house prices would continue to rise. Case, Shiller and Thompson (2012) directly survey homeowners during the boom and find that they expected continued appreciation in house prices over the next decade, as opposed to the bust that eventually occurred. Cheng, Raina and Xiong (2014) find that securitized finance managers did not sell off their personal housing assets during the boom, indicating that these managers had similarly optimistic beliefs relative to the rest of the market. The disagreement our model relies on is in fact consistent with such widespread optimism. Homeowners and investors can be optimistic on average, with dispersion in beliefs around this optimistic mean. Furthermore, only the most optimistic investors price land in the model. Thus, a few extraordinarily optimistic investors have a large price impact, even when nearly all people agree about the future of house prices.

### 5.3 The Cross-Section of Cities

House price increases differed markedly across cities during the 2000-2006 US housing boom. Propositions 1 and 5 derive house price increases as a function of city development levels and disagreement. We test these predictions by interpreting them as comparative statics and then examining them against the empirical variation in house price increases across cities.

Davidoff (2013) analyzes the ability of housing supply elasticities to explain the cross-city variation in house price booms during this time. Part of his analysis measures supply elasticities using price and quantity growth during the 1980s. He finds that many of the largest bubbles (annualized 2000-2007 boom minus annualized 2007-2010 bust) occurred in cities with historically elastic supply and that many of these cities are in the “sand states” of Arizona, California, Florida, and Nevada. We present findings similar to these and then interpret them using our theoretical results.

We use the Federal Housing Finance Agency’s metropolitan statistical area quarterly house price indices, and we measure the housing stock in each city at an annual frequency by interpolating the US Census’s decadal housing stock estimates with its annual housing permit figures. Throughout, we focus on the 115 metropolitan areas for which the population in 2000 exceeds 500,000. The boom consists of the period between 2000 and 2006.

Figure 5(a) plots construction and house price increases across cities. The house price increases vary enormously across cities, ranging from 0% to 125% over this brief six-year period. The largest price increases occurred in two groups of cities. The first group, which we call the Anomalous Cities, consists of Arizona, Nevada, Florida, and inland California; these cities comprise a subset of the “sand state” cities in Davidoff (2013). The other large price increases happened in the Inelastic Cities, which comprise Boston, Providence, New

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Gao, Sockin and Xiong (2015) show that price growth during the boom displays a non-monotonic relationship with respect to Saiz’s (2010) measures of long-run supply elasticity.
York, Philadelphia, and the west coast of the United States; these cities comprise a subset of the “coastal” cities in Davidoff (2013).

The history of construction and house prices in the Anomalous Cities before 2000 constitutes a puzzle for models in which a city’s housing supply elasticity is constant. As shown in Figures 5(b) and 5(c), from 1980 to 2000 these cities provided clear examples of very elastic housing markets in which prices stay low through rapid construction activity. Construction far outpaced the US average while house prices remained constant. In a model like Glaeser, Gyourko and Saiz (2008), in which each city is characterized by a constant housing supply elasticity, the subsequent surge in house prices in these cities is impossible. Perfectly elastic supply should meet whatever housing demand shock arrived in 2000 with higher construction, holding down house prices.

Our model explains this pattern by distinguishing short-run and long-run housing supply elasticities. Proposition 1 shows that prices rise in intermediate cities in which vacant land remains because supply is expected to become constrained soon. This phenomenon depends on the way we have modeled housing supply and holds even without disagreement. Figure 1 demonstrates long-run barriers in Las Vegas. More broadly, the land price increases across the country shown in Figure 3 indicate the presence of these constraints in other cities, or at least developers’ anticipation of them.

The price increases in the Anomalous Cities were as large as those in the Inelastic Cities. The Inelastic Cities consist of markets where house prices rise because regulation and geography prohibit construction from absorbing higher demand. We document this relationship in Figures 5(b) and 5(c), which show that construction in these cities was lower than the US average before 2000 while house price growth greatly exceeded the US average. As shown in Figure 5(a), house prices increased as much in the Anomalous Cities as they did in the Inelastic Cities. Under the assumption of a common demand shock, this pattern poses a puzzle without disagreement: Proposition 1 shows that, without disagreement, constrained cities experience larger house price booms than intermediate and unconstrained cities.

Proposition 5 explains the pattern. With disagreement $z > 0$, the same demand shock $x > 0$ raises the $t = 0$ house price most in intermediate cities, not in constrained cities. According to our model, land availability in the Anomalous Cities facilitated speculation and thus amplified the increase in house prices. This amplification effect was smaller in the Inelastic Cities, which featured less undeveloped land. Evidence of disagreement during the boom comes from the stylized facts about public homebuilders in Section 5.2, as Proposition 7 shows that these facts are guaranteed to hold only with disagreement.

A third puzzle is that some elastic cities built housing quickly during the boom but, unlike the Anomalous Cities, experienced stable house prices. These cities appear in the bottom-right corner of Figure 5(a), and are located mostly in the southeastern United States (e.g.,
Texas and North Carolina). Their construction during the boom quantitatively matches that in the Anomalous Cities, but the price changes are significantly smaller. Why was rapid construction able to hold down house prices in some cities and not others?

Propositions 1 and 5 explain that what distinguishes these cities are their long-run supply elasticities. A city can have perfectly elastic short-run supply, yet its long-run supply is indeterminate. Among the cities with elastic short-run supply, the intermediate cities face constraints soon while the unconstrained cities do not. The model’s explanation of Figure 5(a) is that the Anomalous Cities are the ones approaching the long-run constraints, whereas the cities in the bottom right did not face development barriers in the foreseeable future.

Some evidence consistent with this argument comes from the financial statements of Pulte, one of the homebuilders studied in Section 5.2. In a February 2004 presentation to investors, Pulte listed several of the Anomalous Cities as “supply constrained markets you may not have expected”: West Palm Beach, Orlando, Tampa, Ft. Myers, Sarasota, and Las Vegas (Chicago was also listed). In contrast, Pulte stated that Texas was “the only area of the country without supply constraints in some form,” and listed many of the non-anomalous elastic cities (Atlanta, Charlotte, and Denver) as “not supply constrained overall,” although “supply issues in preferred submarkets” were noted. The slides appear in Appendix E.

An alternate explanation for these cross-sectional patterns is that the Anomalous Cities simply experienced much larger demand growth between 2000 and 2006 than the rest of the country. Abnormally large demand growth would increase prices and construction, leading the Anomalous Cities to occupy the top-right part of Figure 5(a). While the application of our model considers the case of a common demand shock, Figure 5 makes it clear that demand growth did differ across cities. The cluster of cities in the bottom left of the graph likely saw low price growth and construction because demand was flat during this time. The experiences of these cities raise the possibility that the Anomalous Cities saw abnormally large demand growth just as these cities saw abnormally small growth.

25The cities with annual housing stock growth above 2% and cumulative price increases below 25% are Atlanta, Austin, Charlotte, Colorado Springs, Columbus, Dallas, Denver, Des Moines, Fort Collins, Fort Worth, Houston, Indianapolis, Lexington, Nashville, Ogden, Raleigh, and San Antonio.

26Other cities with supply constraints only in submarkets were Phoenix, Jacksonville, Detroit, and Minneapolis.

27The Pulte slides provide narrative support for some of the other assumptions and predictions of the model. Pulte stated that “[Anti-growth efforts] are not new for heavily populated areas (Northeast, California) but now are widespread across the country.” This statement indicates that at least one major developer—the largest public homebuilder at the time—recognized the rise of supply restrictions throughout the country, consistent with our assumption of finite long-run land supply.

28Another explanation is that the value of the option, described by Titman (1983) and Grenadier (1996), to develop land with different types of housing may have been largest in the anomalous cities, but many of these areas consist of homogeneous sprawl (Glaeser and Kahn, 2004), lessening this concern.

29Section 3.4 was silent on the model’s predictions for construction, which we present in Appendix E. The model is ill-suited to explain construction between 2000 and 2006 because it considers only a shock to news about future demand and not a shock to current demand. As shown in Appendix E, the shock $x$ does not
We examine whether any abnormal demand shocks experienced by the Anomalous Cities are sufficient to account for the extreme price movements in these cities. Mian and Sufi (2009) argue that the shock was the expansion of credit to low-income borrowers. It is possible that this shock affected the Anomalous Cities more than the rest of the nation, for instance because they contained greater shares of low income individuals, and that this greater exposure to the shock led to abnormally large price increases.

To address this possibility, we calculate the house price booms that would be predicted from each city’s supply elasticity and relevant demographics in 2000. We construct the predicted price increases in the following manner. Suppose that between 2000 and 2006, each city experienced a permanent increase in log housing demand equal to $x_j$. From Proposition 9, the resulting increase in house prices when $z = 0$ equals

$$
\Delta \log p^h_j = \frac{\mu x_j}{2(\epsilon_s^j + \epsilon)}
$$

to the first order in $x$, where $\epsilon_s^j$ is city $j$’s housing supply elasticity. Because we are exploring a counterfactual without disagreement, this specification assumes that $\mu$ does not vary across cities.

Mian and Sufi (2009) show that the following demographic variables predict the presence of subprime borrowers at the ZIP-code level: household income (negatively), poverty rate, fraction with less than high school education, and fraction nonwhite. We measure these variables at the metropolitan area level in the 2000 US Census, and use them to predict the unobserved shock $x_j$. We denote this vector of demographics, plus a constant and log population, by $d_j$. Under the null hypothesis that these demographics alone predict the shock, we may write $\mu x_j/2 = \beta d_j + \eta_j$, where $\beta$ is the same across cities, and $d_j \perp \eta_j$. Substituting this expression into equation (3) yields the estimating equation

$$(\epsilon_s^j + \epsilon) \Delta \log p^h_j = \beta d_j + \eta_j.$$ 

Estimating $\beta$ using this equation allows us to calculate the house price boom predicted by the supply elasticity $\epsilon_s^j$ and the demographics $d_j$. In equation (4), the left represents the house price increase adjusted by the elasticity of supply, while $\beta d_j$ is the housing demand shock predicted by the city’s exposure to subprime. We use Saiz (2010)’s supply elasticity estimates for $\epsilon_s^j$, and a value of 0.6 for the housing demand elasticity $\epsilon$. This value lies in the range of estimates calculated by Hanushek and Quigley (1980). Using these data, we produce an estimate $\hat{\beta}$ using ordinary least squares on equation (4). The resulting house

\alter the equilibrium housing stock, except in intermediate cities with disagreement where $x > 0$ lowers the stock relative to the case without disagreement.
price boom predicted from demographics and supply elasticity equals

\[
E \left( \Delta \log p^h_j \mid d_j, \epsilon_j^s \right) = \frac{\beta d_j}{\epsilon_j^s + \epsilon}.
\]

Figure 6 plots the actual house price growth against the predicted price growth for each city in Figure 5(a). The Anomalous Cities remain clear outliers.\(^{30}\) Abnormal demand growth from low-income borrowers does not explain the extreme experiences of these cities. In theory, these predicted price increases could have lined up well with the actual increases in the Anomalous Cities. This alignment would have held if the subprime demographics predicted the shocks, these cities were very exposed to subprime, and their housing supply elasticities were low enough. This story fails to explain the anomalous house price booms, which experienced higher price growth despite elastic supply and even conditioning on observable drivers of demand. Furthermore, the growth in subprime credit was widespread, with high-housing supply elasticity cities experiencing large expansions in subprime credit without house price growth (Mian and Sufi, 2009, Table VII).

5.4 Variation in House Price Booms Within Cities

Proposition 8 of the model predicts larger price increases in market segments within a city that attract more renters than owners. A sufficient statistic for this effect is \(\chi\), the share of the housing stock that is rented. Proposition 8 holds only among segments with the same \(N_0, x\) and \(z\). This “all else equal” assumption is unlikely to hold empirically, so our discussion focuses on the conceptual predictions about within-city variation made by the model.

We first consider variation in \(\chi\) across neighborhoods. Neighborhoods provide an example of market segments because they differ in the amenities they offer. For instance, some areas offer proximity to restaurants and nightlife, while others provide access to good public schools. These amenities appeal to different groups of residents. Variation in amenities hence leads \(\chi\) to vary across space. Neighborhoods whose amenities appeal relatively more to renters than to owner-occupants are characterized by a higher value of \(\chi\).

We obtain ZIP-level data on \(\chi\) from the US Census, which reports the share of occupied housing that is rented, as opposed to owner-occupied, in each ZIP code in 2000. \(\chi\) varies considerably within cities. Its national mean is 0.29 and standard deviation is 0.17, while the \(R^2\) of regressing \(\chi\) on city fixed-effects is only 0.12. We calculate the real increase in house prices from 2000 to 2006 using Zillow.com’s ZIP-level house price indices. We regress this price increase on \(\chi\) and city fixed-effects, and find a positive and highly significant coefficient of 0.10 (0.026), where the standard error is clustered at the city level. Thus, consistent with

\(^{30}\)Appendix Table E1 presents regression coefficients corresponding to Figure 6. The table shows that the degree to which the Anomalous Cities are outliers is only slightly smaller when demographics are controlled.
Proposition 8, house prices increased more between 2000 and 2006 in neighborhoods where \( \chi \) was higher in 2000.

This positive relationship between \( \chi \) and price increases may not be causal. Housing demand shocks in the boom were larger in neighborhoods with a higher value of \( \chi \). The housing boom resulted from an expansion of credit to low-income households (Mian and Sufi, 2009; Landvoigt, Piazzesi and Schneider, 2015). As a result, the strong covariance of ZIP-level income with \( \chi \) will tend to bias our estimates.\(^{31}\) Furthermore, a city-wide demand shock might raise house prices most strongly in cheap areas due to gentrification dynamics (Guerrieri, Hartley and Hurst, 2013), and \( \chi \) covaries negatively with the level of house prices within a city.

The appeal of \( \chi \) is that it predicts price increases in any housing boom in which there is disagreement about future fundamentals. In general, \( \chi \) predicts price increases because it is positively correlated with speculation, not because it is correlated with demand shocks. Empirical work can test Proposition 8 by examining housing booms in which the shocks are independent from \( \chi \).

The second approach to measuring \( \chi \) is to exploit variation across different types of housing structures. According to the US Census, 87% of occupied detached single-family houses in 2000 were owner-occupied rather than rented. In contrast, only 14% of occupied multifamily housing was owner-occupied. According to Proposition 8, the large difference in \( \chi \) between these two types of housing causes a larger price boom in multifamily housing, all else equal.

This result squares with accounts of heightened investment activity in multifamily housing during the boom.\(^{32}\) For instance, a consortium of investors—including the Church of England and California’s pension fund CalPERS—purchased Stuyvesant Town & Peter Cooper Village, Manhattan’s largest apartment complex, for a record price of $5.4 billion in 2006. Their investment went into foreclosure in 2010 as the price of this complex sharply fell (Segel et al., 2011). Multifamily housing attracts speculators because it is easier to rent out than single-family housing. During periods of uncertainty, optimistic speculators bid up multifamily house prices and cause large price booms in this submarket.

\section{Conclusion}

In this paper, we argue that disagreement explains an important part of housing cycles. Disagreement amplifies house price booms by biasing prices toward optimistic valuations.\(^{31}\) The IRS reports the median adjusted gross income at the ZIP level. We take out city-level means, and the resulting correlation with \( \chi \) is \(-0.40\).\(^{32}\) Bayer et al. (2015) develop a method to identify speculators in the data. A relevant extension of their work would be to look at the types of housing speculators invest in.

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Our emphasis on how disagreement interacts with long run development constraints allows us to explain aspects of the boom that are at odds with existing theories of house prices. Many of the largest price increases occurred in cities that were able to build new houses quickly. This fact poses a problem for theories that stress inelastic housing supply as the sole source of house price booms. But it sits well with our theory, which instead emphasizes speculation. Undeveloped land facilitates speculation due to rental frictions in the housing market. In our model, large price booms occur in elastic cities facing a development barrier in the near future.

Introducing key aspects of the housing market—heterogeneous ownership utility and the nature of asset supply—extends and clarifies past work in finance that has focused on disagreement in financial asset markets. In our model, disagreement raises the price of housing only under certain conditions and the relationship between disagreement and asset supply can be non-monotonic. The particular setting of housing markets also presents an important case in which disagreement reduces welfare, as pessimists with high flow utility may be replaced by optimists with low flow utility.

We document the central importance of land price increases for explaining the US house price boom between 2000 and 2006. These land price increases resulted from speculation directly in the land market. Consistent with this theory, homebuilders significantly increased their land investments during the boom and then suffered large capital losses during the bust. Many investors disagreed with this optimistic behavior and short-sold homebuilder equity as the homebuilders were buying land.

In one of the model's extensions, price booms are larger in submarkets within a city where a greater share of housing is rented. We present some evidence for this prediction, but further empirical work is needed to test it more carefully. We also look forward to work exploring these findings to understand cycles outside the US, in historical episodes, and in other markets with similar features to housing.
References


FIGURE 1
Long-Run Development Constraints in Las Vegas

Notes: This figure comes from page 51 of the Regional Transportation Commission of Southern Nevada’s Regional Transportation Plan 2009-2035 (RTCSNV, 2012). The first three pictures display the Las Vegas metropolitan area in 1980, 1990, and 2008. The final picture represents the Regional Transportation Commission’s forecast for 2030. The boundary is the development barrier stipulated by the Southern Nevada Public Land Management Act. The shaded gray region denotes developed land.
FIGURE 2
House Price Boom for Different Initial Demands

Notes: The boom size equals $p_h^b(N_0, x, z)/p_h^b(N_0, 0, z) - 1$ with disagreement and $p_h^b(N_0, x, 0)/p_h^b(N_0, 0, 0) - 1$ without disagreement. $N_0$ equals the number of potential residents at $t = 0$ relative to the amount of space in the city. The parameter values used to generate this figure are $x = 0.5$, $z = 1$, $\epsilon = 1$, $\mu = 0.2$, and $f_r = f_d$ with 90 percent of agents having $\theta = -1/9$ and 10 percent having $\theta = 1$. These parameters are defined in Section 1.
FIGURE 3
Input Price and House Price Increases Across Cities, 2000-2006

Notes: We measure construction costs for each city using the R.S. Means survey figures for the marginal cost of a square foot of an average quality home, deflated by the CPI-U. Gyourko and Saiz (2006) contains further information on the survey. Land price changes come from the hedonic indices calculated in Nichols, Oliner and Mulhall (2013) using land parcel transactions, and house prices come from the second quarter FHFA housing price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data. For land prices, we have data for Atlanta, Baltimore, Boston, Chicago, Dallas, Denver, Detroit, Houston, Las Vegas, Los Angeles, Miami, New York, Orlando, Philadelphia, Phoenix, Portland, Sacramento, San Diego, San Francisco, Seattle, Tampa, and Washington D.C.
FIGURE 4
Supply-Side Speculation Among U.S. Public Homebuilders, 2001-2010

a) Land Holdings and Home Sales

b) Market Equity

c) Short Interest

Notes: (a), (b) Data come from the 10-K filings of Centex, Pulte, Lennar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific, the eight largest public U.S. homebuilders in 2001. “Lots Controlled” equals the sum of lots directly owned and those controlled by option contracts. The cumulative writedowns to land holdings between 2006 and 2010 among these homebuilders totals $29 billion. (c) Short interest is computed as the ratio of shares currently sold short to total shares outstanding. Monthly data series for shares short come from COMPUSTAT and for shares outstanding come from CRSP. We compute mean short interest between 2000 and 2006 for each six-digit NAICS industry and plot the cumulative distribution of these means. Builder stocks are classified as those with NAICS code 236117 and investment bank stocks are those with NAICS code 523110.
FIGURE 5
The U.S. Housing Boom and Bust Across Cities

a) Price Increases and Construction, 2000-2006

![Graph showing the relationship between cumulative price increase and annual housing stock growth across different cities.]

b) Historic Construction
![Graph showing the annual housing stock growth over time for anomalous cities, U.S. average, and inelastic cities.]

c) Historic Prices
![Graph showing the cumulative price increase over time for anomalous cities, U.S. average, and inelastic cities.]

Notes: Anomalous Cities include those in Arizona, Nevada, Florida, and inland California. Inelastic Cities are Boston, Providence, New York, Philadelphia, and all cities on the west coast of the United States. We measure the housing stock in each city at an annual frequency by interpolating the U.S. Census’s decadal housing stock estimates with its annual housing permit figures. House price data come from the second quarter FHFA house price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data. (a) The cumulative price increase is the ratio of the house price in 2006 to the house price in 2000. The annual housing stock growth is the log difference in the housing stock in 2006 and 2000 divided by six. (b), (c) Each series is an average over cities in a group weighted by the city’s housing stock in 2000. Construction is annual permitting as a fraction of the housing stock. Prices represent the cumulative returns from 1980 on the housing in each group.

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FIGURE 6
Anomalous Cities and Differential Demand Shocks, 2000-2006

Notes: This plot compares actual price growth during the boom to predicted price growth as a function of city level demographics, where predicted price growth proxies for differential demand shocks. Actual price growth is the log change in the second quarter FHFA house price index deflated by the CPI-U. We compute predicted price growth from a cross sectional regression of actual price growth on a set of city level demographics: log population, log of median household income, percent white, percent white and not hispanic, percent with less than 9th grade education, percent with less than 12th grade education, percent unemployed, and percent of families under the poverty line. Demographics come from the 2000 Census.
A Proofs

Lemma 1

If \( p^h_1 > p^l_1 + k \), then each developer wants to buy an infinite amount of land, build houses with the land, and then sell the houses. As a result, the land market cannot clear. If \( p^h_1 < p^l_1 + k \), then the reverse holds, meaning that developers want to sell an infinite amount of land. The land market cannot clear in this case either. In equilibrium, the only possibility is that \( p^h_1 = p^l_1 + k \).

At \( t = 1 \), demand from arriving potential residents equals \( N_1 S D(p^l_1) \). Supply from outgoing residents equals 0 if \( p^h_1 < 0 \) and \( Q_r \) if \( p^h_1 > 0 \), where \( Q_r \) is the number of potential residents who bought at \( t = 0 \). Developers are indifferent to how much housing they sell because \( p^h_1 = p^l_1 + k \), but the most they can sell emerges from summing across the two developer constraints to obtain \( \sum H^s_1 \leq \sum H_1 + L_1 = S - Q_r \). The sum of \( H^s_1 \) across developers and potential residents cannot exceed \( S \).

We now consider three possible equilibria. In the first, \( p^l_1 < 0 \). This inequality cannot hold in equilibrium because developer land demand would be infinite for each developer, and the land market would not clear. The next possibility is that \( p^l_1 = 0 \). In this case, demand from arriving potential residents equals \( N_1 S \). If \( N_1 > 1 \), then this equilibrium fails because maximal aggregate home sales equal \( S \). If \( N_1 \leq 1 \), then we construct an equilibrium as follows. We cannot have \( N_1 S > L_1 + H_1 + Q_r \) for all developers (summing across them delivers a contradiction when \( N_1 \leq 1 \)), so consider a developer for whom \( N_1 S \leq L_1 + H_1 + Q_r \). This developer sets \( L_1^{buy} = 0 \), \( H_1^{build} = N_1 S - Q_r - H_1 \), and \( H_1^{sell} = N_1 S - Q_r \). All other developers set \( L_1^{buy} = 0 \), \( H_1^{build} = -H_1 \), and \( H_1^{sell} = 0 \). All developer constraints and optimality conditions are satisfied under these choices, and both the housing and land markets clear. Finally, we consider the possibility that \( p^l_1 > 0 \). Because \( p^l_1 > 0 \), the first constraint for the developers binds, so we can rewrite the developer objective function as \( p^h_1 H_1 + p^l_1 (H_1^{build} - L_1^{buy}) \). Because \( p^l_1 > 0 \), the developer maximizes this objective by satisfying the second constraint and setting \( H_1^{build} = L_1^{buy} = L_1 \). Because both constraints are satisfied with equality, summing across them yields \( \sum H_1^{sell} = \sum H_1 + \sum L_1 + \sum L_1^{buy} \). If the land market clears, the last sum equals 0. The housing market clears when \( N_1 S D(p^h_1) = \sum H_1 + \sum L_1 + Q_r = S \). If \( N_1 < 1 \), then no solution for \( p^h_1 \) exists. If \( N_1 = 1 \), then the only solutions have \( p^h_1 \leq k \), contradicting the assumption that \( p^l_1 > 0 \). This equilibrium is possible if and only if \( N_1 > 1 \), in which case the unique solution is \( p^h_1 = k N_1^{1/\epsilon} \). The optimality conditions and constraints for all developers are satisfied if they set \( H_1^{build} = L_1 \), \( H_1^{sell} = H_1 + L_1 \), and \( L_1^{buy} = 0 \). The land and housing markets clear under these choices. In summary, a unique equilibrium exists for each value of \( N_1 \). When \( N_1 \leq 1 \), only \( p^l_1 = 0 \) and \( p^h_1 = k \) are possible, whereas when \( N_1 > 1 \), only \( p^l_1 = k N_1^{1/\epsilon} - k \) and \( p^h_1 = N_1^{1/\epsilon} \) are possible.

Lemma 2

The utility at \( t = 1 \) of a resident who bought at \( t = 0 \) equals \( p^h_1 - p^h_0 + v \) if \( p^h_0 \geq 0 \). Housing demand from potential residents at \( t = 0 \) equals \( \int_\Theta N_0 S D(p^l_0 - p^l_1 (e^{\mu(\theta)x} N_0)) f_\theta(\theta) \, d\theta \). For the same argument given in the proof of Lemma 1 that \( p^h_1 = p^l_1 + k \), \( p^l_0 = p^l_0 + 2k \) in equilibrium: developers would want to buy or sell infinite land otherwise. In all of the equilibria characterized in the proof of Lemma 1, \( \pi = p^h_1 H_1 + p^l_1 L_1 + B_1 \). By making substitutions using the constraints of the \( t = 0 \) developer problem, we see that the objective at \( t = 0 \) is to choose \( H_1, L_1 \) \( \geq 0 \) to maximize \( (p^l_1 (e^{\mu(\theta)x} N_0) - p^h_0) H_1 + (p^l_1 (e^{\mu(\theta)x} N_0) - p^h_0 + k) L_1 + p^l_0 L_0 \). In all equilibria, all developers choose finite values of \( H_1 \) and \( L_1 \), so the first order conditions imply \( p^h_1 (e^{\mu(\theta)x} N_0) - p^h_0 + k \leq 0 \) for all
\( \theta \in \text{supp } f_d. \) Because \( p^h_0(\cdot) \) increases, either \( p^h_0 = p^h_0(\mu_{d, \text{max}} N_0) + k \) in which case developers with \( \theta = \theta^*_{d, \text{max}} \) may choose any \( L_1 \geq 0, \) or \( p^h_0 > p^h_1(\mu_{d, \text{max}} N_0) + k \) in which case \( L_1 = 0 \) for all developers.

We now consider these two possible equilibria. The first may hold only if potential resident housing demand does not exceed \( S \) (developers cannot build more than this quantity of housing, and the housing market must clear). This condition reduces to

\[
S \theta = \text{max } \text{housing demand does not exceed } p \theta
\]

Proposition 2 case, if \( \text{equation in the proposition follows from the solution for } p \theta \)

Because \( L^\text{build} = Q_r, H^\text{sell} = Q_r, \) and \( L^\text{buy} = S - L_0. \) For all other developers, we set \( H^\text{build} = 0, H^\text{sell} = 0, \) and \( L^\text{buy} = -L_0. \) All developer constraints and optimality conditions hold, and the housing and land markets clear.

If (A1) fails, then we must have \( N_0 > 0 \) because \( D \leq 1. \) As a result, we may define \( p^h_0(N_0, x, z) \) to be the unique solution to

\[
1 = N_0 \int_\Theta D(p^h_1(e^{\mu_{d, \text{max}} x} N_0) + k - p^h_1(\mu(\theta)x N_0)) f_r(\theta) d\theta.
\]

(A1)

If (A1) holds, then we construct an equilibrium as follows. Denote \( Q_r = \int_\Theta N_0 SD(p^h_1(e^{\mu_{d, \text{max}} x} N_0) + k - p^h_1(\mu(\theta)x N_0)) f_r(\theta) d\theta. \) For one developer for whom \( \theta = \theta^*_{d, \text{max}}, \) we set \( H^\text{build} = Q_r, H^\text{sell} = Q_r, \) and \( L^\text{buy} = S - L_0. \) For all other developers, we set \( H^\text{build} = 0, H^\text{sell} = 0, \) and \( L^\text{buy} = -L_0. \) All developer constraints and optimality conditions hold, and the housing and land markets clear.

To see that a solution to (A2) exists, consider that when \( N_0 > 0 \) (pointwise) as \( p^h_0 \rightarrow \infty, \) by the intermediate value theorem we may find \( p^h_0 \) satisfying (A2). This solution is unique because the integrand strictly decreases if \( p^h_0 - p^h_1(\mu(\theta)x N_0) \geq k; \) this equation must hold at \( p^h_0 = p^h_0(N_0, x, z) \) for at least some \( \theta \in \text{supp } f_r \) for otherwise the right side of (A2) exceeds 1. Because the right side of (A2) weakly decreases in \( p^h_0, \) if (A1) fails then \( p^h_0(N_0, x, z) > p^h_1(\mu_{d, \text{max}} N_0) + k. \) As a result, developer constraints and optimality conditions are satisfied when for all developers \( L^\text{buy} = 0, H^\text{build} = L_0, \) and \( H^\text{sell} = L_0; \) housing and land markets clear as well.

In summary, if (A1) holds then the unique equilibrium price is \( p^h_0(N_0, x, z) = p^h_1(\mu_{d, \text{max}} N_0) + k; \) if (A1) fails, then the unique equilibrium price \( p^h_0(N_0, x, z) \) is the unique solution to (A2).

Proposition 1

From the proof of Lemma 2, developers hold at the end of \( t = 0 \) if and only if (A1) holds without equality. When \( \theta = 0, \mu(\theta) = \mu_{d, \text{max}} \) for all \( \theta \in \Theta \) so (A1) reduces to \( 1 \geq N_0. \) As a result, developers hold land at the end of \( t = 0 \) if and only if \( N_0 < 1, \) as claimed. In this case, \( p^h_0(N_0, x, 0) = p^h_1(\mu_{d, \text{max}} N_0) + k. \) If \( N_0 < e^{-\bar{\mu} x}, \) then (from Lemma 1) \( p^h_0(N_0, x, 0) = 2k; \) if \( e^{-\bar{\mu} x} \leq N_0 < 1, \) then \( p^h_0(N_0, x, 0) = k(e^{\bar{\mu} x} N_0^{1/\epsilon} - 1). \) If \( N_0 \geq 1, \) then \( p^h_0(N_0, x, 0) = k N_0^{1/\epsilon} + p_1(e^{\bar{\mu} x} N_0) \) is the unique solution to (A2). Because \( \bar{\mu} x \geq 0, \) in this case \( p^h_0(N_0, x, 0) = k(1 + e^{\bar{\mu} x/\epsilon}) N_0^{1/\epsilon} \), as claimed. The final equation in the proposition follows from the solution for \( p^h_0(N_0, x, 0); \) note that in the intermediate case, if \( x = 0, \) then \( p^h_0 = 2k \) as in the unconstrained case.

Proposition 2

The number of potential residents of type \( \theta \) who purchase housing equals \( N_0 SD(p^h_0(N_0, x, z) - p_1(\mu(\theta)x N_0)) > 0, \) so Assumptions 1 and 2 guarantee that residents of each type \( \theta \in \text{supp } f_r \) hold housing. To prove the other parts of the proposition, we show that there exists a unique \( N^*_0(x, z) \in \mathbb{R}_{>1} \cup \{\infty\} \) such that (A1) holds strictly if and only if \( N_0 < N^*_0(x, z) \) and with equality
if and only if \( N_0 = N^*_0(x, z) \). We may rewrite (A1) as

\[
1 \geq \begin{cases} 
N_0 & \text{if } N_0 \leq e^{-\mu_d^{maxx}}x \\
\int_{\Theta_1(N_0, x, z)} N_0 f_r(\theta) d\theta + \int_{\Theta_2(N_0, x, z)} e^{-\mu_d^{maxx}} f_r(\theta) d\theta \\
+ \int_{\Theta_3(N_0, x, z)} \left(N_0^{-1/\epsilon} + e^{\mu_d^{maxx}/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon} f_r(\theta) d\theta & \text{if } N_0 > e^{-\mu_d^{maxx}},
\end{cases}
\]

where \( \Theta_1(N_0, x, z) = \{ \theta \mid \theta \geq \theta_d^{max} \} \), \( \Theta_2(N_0, x, z) = \{ \theta \mid \theta < \theta_d^{max} \} \cap \{ \theta \mid e^{\mu(\theta)x} N_0 \leq 1 \} \), and \( \Theta_3(N_0, x, z) = \{ \theta \mid \theta < \theta_d^{max} \} \cap \{ \theta \mid e^{\mu(\theta)x} N_0 > 1 \} \). For notational ease, we name the right side of this inequality \( \phi(N_0) \). We have \( \lim_{N_0 \to 0} \phi(N_0) = 0 \), and \( \phi \) strictly increases for \( 0 < N_0 \leq e^{-\mu_d^{maxx}}x \). The integrands coincide for \( \theta \) in the boundary of \( \Theta_2 \) and \( \Theta_3 \), so \( \phi(N_0) \) for \( N_0 \geq e^{-\mu_d^{maxx}}x \) equals the sum of the derivatives under the integral signs (the changing limits of integration cancel out). Therefore \( \phi \) strictly increases in \( N_0 \) for all \( N_0 > 0 \) except those for which \( \Theta_2(N_0, x, z) = \text{supp} f_r \). For any such \( N_0 \), \( 1 > \phi(N_0) \) because \( \mu_d^{max} > \mu \geq 0 \) given Assumption 4 and given that \( z > 0 \). The increasing nature of \( \phi \) means that there may exist at most one solution to \( 1 = \phi(N_0) \), and that (A1) is satisfied strictly for any \( N_0 \) less than this solution and is not satisfied for any \( N_0 \) greater than this solution. We dedem the solution \( N_0^*(x, z) \). Note that \( \phi(1) < 1 \) unless \( \int_{\Theta_1(x, z)} f_r(\theta) d\theta = 1 \), which is impossible by Assumption 4. Therefore \( \phi(1) < 1 \) and \( N_0^*(x, z) > 1 \). For later proofs, we note here that \( \lim_{z \to 0} N_0^*(x, z) = 1 \), which is evident because for any \( N_0 > 1 \), \( \lim_{z \to 0} \phi(N_0) > 1 \), while \( \phi(1) < 1 \) for all \( z > 0 \).

The existence of \( N_0^*(x, z) \) implies that some developers hold land if and only if \( N_0 < N_0^*(x, z) \). If \( N_0 < N_0^*(x, z) \), then the proof of Lemma 2 shows that \( p^h_0 = p^h_1(e^{\mu_d^{maxx}}x N_0) + k \) and that a developer of type \( \theta \) may hold land if and only if \( p^h_1(e^{\mu(\theta)x} N_0) + k \leq p^h_0(N_0, x, z) \). If \( N_0 \leq e^{-\mu_d^{maxx}}x \), then \( p^h_1(e^{\mu(\theta)x} N_0) = k \) for all \( \theta \in \text{supp} f_d \), so any developer may hold land at the end of \( t = 0 \), as claimed. If \( e^{-\mu_d^{maxx}}x < N_0 < N_0^*(x, z) \), then the only developers for whom \( p^h_1(e^{\mu(\theta)x} N_0) \leq p^h_0(e^{\mu_d^{maxx}}x N_0) \) are those for \( \theta = \theta_d^{max} \) due to Lemma 1. As a result, only these developers hold land when \( N_0 \) satisfies these constraints, as claimed.

We now prove the result on excess land holdings by developers in the intermediate case. Define \( Q_r \) to be the quantity of housing held by potential residents in equilibrium. From Lemma 2, \( Q_r \) does not depend on the developer land endowments \( L_0 \), and by the first part of the proposition, \( 0 < Q_r < S \) for \( e^{-\mu_d^{maxx}}x < N_0 < N_0^*(x, z) \). Summing across the constraint on \( L_1 \) for developers at \( t = 0 \) with \( \theta = \theta_d^{max} \) yields \( S - Q_r = \sum L_1 = \sum (L_0 + (L_0^{bay})^* - (H_0^{build})^*) \). As a result, \( \sum ((L_0^{bay})^* - (H_0^{build})^*) = S - Q_r = \sum L_0 \), which exceeds zero as long as \( \sum L_0 < S - Q_r = L^* \).

Finally, if \( \int_{\theta > \theta_d^{max}} f_r(\theta) d\theta > 0 \), then \( \lim_{N_0 \to \infty} \phi(N_0) = \infty \), leading to \( N_0^*(x, z) < \infty \). If \( \int_{\theta > \theta_d^{max}} f_r(\theta) d\theta = 0 \), then \( \lim_{N_0 \to \infty} \phi(N_0) = \int_{\theta > \theta_d^{max}} (e^{\mu_d^{maxx}/\epsilon} - e^{\mu(\theta)x/\epsilon}) f_r(\theta) d\theta \), so \( N_0^*(x, z) < \infty \) in this case if and only if this integral exceeds 1.

**Lemma 3**

First we prove existence and uniqueness of \( \mu^{agg}_r(N_0, x, z) \). For \( N_0 \geq N_0^*(x, z) \), \( p^h_0(N_0, x, z) \geq p^h_1(e^{\mu_d^{maxx}}x N_0) + k \) as shown in the proof of Lemma 2. Because \( \mu_d^{max} > \mu \geq 0 \), \( p^h_1(e^{\mu_d^{maxx}}x N_0) = k e^{\mu_d^{maxx}/\epsilon} N_0^{1/\epsilon} \), so \( p^h_0(N_0, x, z) > k N_0^{1/\epsilon} \). It follows that \( p^h(N_0, x, z) = k(1 + e^{\mu_d^{agg}}(N_0, x, z)x/\epsilon)) N_0^{1/\epsilon} \) has a unique solution for \( \mu^{agg}_r(N_0, x, z) \), as the right side strictly increases from \( k N_0^{1/\epsilon} \) to \( \infty \) as \( \mu^{agg}_r(N_0, x, z) \) goes to \( \infty \) (which holds due to the assumption that \( x > 0 \)).

We next show that \( \mu^{agg}_r(N_0, x, z) \leq \mu^{max} \). Because the right side of (2) weakly decreases in
\( \mu_r^{agg} \), it suffices to show that
\[
1 > N_0 \int_\Theta D(k(1 + e^{\mu_{r}^{max}x/\epsilon})N_0^{1/\epsilon} - p_h^1(e^{\mu(x)}N_0))f_r(\theta)d\theta. \quad (A3)
\]
Due to Assumption 4, \( \mu_{r}^{max} > 0 \) and \( \int_{\Theta} f_r(\theta)d\theta > 0 \). Because \( p_h^1(\cdot) \) strictly increases on \([1, \infty)\), \( p_h^1(e^{\mu(x)}N_0) - p_h^1(e^{\mu_{r}^{max}x}N_0) \) for \( \theta < \theta_{r}^{max} \). When \( \theta = \theta_{r}^{max} \), the argument of \( D(\cdot) \) in (A3) equals \( kN_0^{1/\epsilon} \). Because \( D(\cdot) \) strictly decreases on \([k, \infty)\), it follows that the right side of (A3) is strictly less than \( N_0D(kN_0^{1/\epsilon}) = 1 \), as desired.

We next use the conditions in the lemma to simplify (2). Because \( p_h^1(N_0, x, z) \geq p_h^1(e^{\mu_{r}^{max}x}N_0) + k \) for \( N_0 \geq N_0^*(x, z) \) (as shown in the proof of Lemma 2), for all \( \theta \in \supp f_r \), we may bound the argument of \( D(\cdot) \) in (2) as follows: \( (1 + e^{\mu_{r}^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} - p_h^1(e^{\mu(x)}N_0) \geq k + p_h^1(e^{\mu_{r}^{max}x}N_0) \), with the last inequality holding because \( \theta \leq \theta_{r}^{max} \) by assumption. Furthermore, because \( \theta \geq -p/z \) for all \( \theta \in \supp f_r, \mu(\theta) \geq 0 \) for all such \( \theta \), allowing us to write \( p_h^1(e^{\mu(x)}N_0) = ke^{\mu(x)/N_0^{1/\epsilon}} \) for all \( \theta \in \supp f_r \). It follows from Assumption 1 that \( \mu_r^{agg}(N_0, x, z) \) must solve
\[
1 = \int_{\supp f_r} \left( 1 + e^{\mu_{r}^{agg}(N_0,x,z)x/\epsilon} - e^{\mu(x)x/\epsilon} \right)^{-\epsilon} f_r(\theta)d\theta. \quad (A4)
\]
We finally show that \( \mu_r^{agg}(N_0, x, z) \geq \overline{\mu} \) and that \( \mu_r^{agg}(N_0, x, z) = \overline{\mu} + o(z) \) as \( z \to 0 \). The argument of the integral in (A4) is convex in \( \theta \) for \( \theta \in \supp f_r \), so Jensen’s inequality implies that \( 1 > (1 + e^{\mu_{r}^{agg}(N_0,x,z)x/\epsilon} - e^{\mu(x)x/\epsilon})^{-\epsilon} \), from which it follows that \( \mu_r^{agg}(N_0, x, z) > \overline{\mu} \). Taking the derivative of (A4) with respect to \( z \) and simplifying yields
\[
\frac{\partial \mu_r^{agg}(N_0, x, z)}{\partial z} = \frac{\int_{\supp f_r} \theta e^{\mu(x)/x}(1 + e^{\mu_{r}^{agg}(N_0,x,z)x/\epsilon} - e^{\mu(x)x/\epsilon})^{-1} f_r(\theta)d\theta}{\int_{\supp f_r} e^{\mu_{r}^{agg}(N_0,x,z)x/\epsilon}(1 + e^{\mu_{r}^{agg}(N_0,x,z)x/\epsilon} - e^{\mu(x)x/\epsilon})^{-1} f_r(\theta)d\theta}.
\]
As \( z \to 0 \), the denominator goes to \( e^{\mu(x)x/\epsilon} \), whereas the numerator goes to \( \int_{\Theta} e^{\mu(x)/x} f_r(\theta)d\theta \), which equals 0 because \( \int_{\Theta} \theta f_r(\theta)d\theta = 0 \) by assumption. Therefore \( \mu_r^{agg}(N_0, x, z) = \overline{\mu} + o(z) \) as \( z \to 0 \).

**Proposition 3**
If \( N_0 < N_0^*(x, z) \), then (A1) holds and \( p_h^1(N_0, x, z) = p_h^1(e^{\mu_{r}^{max}x}N_0) + k \). By applying Lemma 1, we arrive at the first two pricing equations in Proposition 3. The equation for \( N \geq N_0^*(x, z) \) follows immediately from (A2).

**Proposition 4**
Substituting the formulas for \( p_h^1(N_0, x, z) \) from Proposition 2 and for \( p_h^1(N_0, x, 0) \) from Proposition 1 yield the equations in the first part of Proposition 4. For clarity, we prove the remainder of the claims in Proposition 4 in three parts.

*Part 1: Disagreement effect is maximized at \( N_0 = 1 \)*

The effect of disagreement on the house price at \( t = 0 \) weakly increases in \( N_0 \) up to \( N_0 = 1 \) and decreases for \( 1 \leq N_0 \leq N_0^*(x, z) \). The maximum for \( 0 < N_0 < N_0^*(x, z) \) therefore equals the value at \( N_0 = 1 \), which is \( (e^{\mu_{r}^{max}x/\epsilon} - e^{\mu(x)/\epsilon})/(1 + e^{\mu(x)/\epsilon}) \). This value exceeds the disagreement effect for all \( N_0 \geq N_0^*(x, z) \) if and only if \( \mu_{r}^{max} > \mu_r^{agg}(N_0, x, z) \) for all \( N_0 \geq N_0^*(x, z) \). We prove this in two steps. We first show that \( p_h^1(N_0, x, z) \), and hence \( \mu_r^{agg}(N_0, x, z) \), weakly increases in \( N_0 \) for
\[ N_0 \geq N_0^*(x, z) \]. Second, to show that \( \mu^{agg}_r(N_0, x, z) < \mu^{\max}_d \) for all \( N_0 \geq N_0^*(x, z) \), we show that \( \lim_{N_0 \to \infty} \mu^{agg}_r(N_0, x, z) \) exists and is less than \( \mu^{\max}_d \).

We may rewrite (A2) as

\[ 1 = \int_{\Theta_1(N_0, x, z)} N_0 D(p_0^h - k)f_r(\theta)d\theta + \int_{\Theta_2(N_0, x, z)} N_0 \left( \frac{k}{p_0^h - ke^{\mu(x)/\epsilon}N_0^{1/\epsilon}} \right)^{\epsilon}f_r(\theta)d\theta + \int_{\Theta_3(N_0, x, z)} N_0 f_r(\theta)d\theta, \]

where \( \Theta_1(N_0, x, z) = \{ \theta \mid e^{\mu(x)/\epsilon}N_0 < 1 \} \), \( \Theta_2(N_0, x, z) = \{ \theta \mid 1 \leq e^{\mu(x)/\epsilon}N_0 \leq (p_0^h/k - 1)^{\epsilon} \} \), and \( \Theta_3(N_0, x, z) = \{ \theta \mid e^{\mu(x)/\epsilon}N_0 > (p_0^h/k - 1)^{\epsilon} \} \). For a given \( p_0^h \), the right side of (A5) weakly increases in \( N_0 \); each integrand weakly increases in \( N_0 \) (for each \( \theta \)), and the integrands coincide at the boundaries of the limits of integration, meaning that the marginal effect from changing the limits of integration equals 0. Because the right side of (A2) weakly decreases in \( p_0^h \) (as shown in the proof of Lemma 2), it follows that \( p_0^h(N_0, x, z) \) weakly increases in \( N_0 \).

This monotonicity means that for all \( N_0 \geq N_0^*(x, z) \), \( \mu^{agg}_r(N_0, x, z) \leq \lim_{N_0' \to \infty} \mu^{agg}_r(N_0', x, z) \).

Substituting \( p_0^h(N_0, x, z) = k(1 + e^{\mu^{agg}(N_0, x, z)/\epsilon})N_0^{1/\epsilon} \) into (A5) yields

\[ 1 = \int_{\Theta_1(N_0, x, z)} (1 + e^{\mu^{agg}(N_0, x, z)/\epsilon} - N_0^{-1/\epsilon})^{-\epsilon}f_r(\theta)d\theta + \int_{\Theta_2(N_0, x, z)} (1 + e^{\mu^{agg}(N_0, x, z)/\epsilon} - e^{\mu(x)/\epsilon})^{-\epsilon}f_r(\theta)d\theta + \int_{\Theta_3(N_0, x, z)} N_0 f_r(\theta)d\theta. \]

Because \( \mu^{agg}(N_0, x, z) \) increases in \( N_0 \), \( \lim_{N_0 \to \infty} \mu^{agg}_r(N_0, x, z) \) either exists and is finite or it equals \( \infty \). In the latter case, because \( \mu(\theta) \leq \mu^{\max}_r \) for all \( \theta \in supp f_r \), each integral goes to 0 as \( N_0 \to \infty \), leading to a contradiction. (That the last integral \( \to 0 \) follows because \( \theta \in \Theta_3(N_0, x, z) \) if and only if \( e^{\mu(x)/\epsilon}N_0 \geq ((1 + e^{\mu^{agg}(N_0, x, z)/\epsilon})N_0^{1/\epsilon} - 1)^{\epsilon} \), which implies that \( \mu(\theta) \geq \mu^{agg}_r(N_0, x, z) \) because \( N_0 \geq N_0^*(x, z) > 1 \). For all \( N_0 \) such that \( \mu^{agg}_r(N_0, x, z) > \mu^{\max}_r \), \( \int_{\Theta_3(N_0, x, z)} f_r(\theta)d\theta = 0 \). Thus, \( \lim_{N_0 \to \infty} \mu^{agg}_r(N_0, x, z) < \infty \). In this case, \( \lim_{N_0 \to \infty} \Theta_3(N_0, x, z) = \{ \theta \mid e^{\mu(x)/\epsilon} \geq 1 + e^{\lim_{N_0 \to \infty} \mu^{agg}_r(N_0, x, z)/\epsilon} \} \), whose measure under \( f_r \) must equal 0 for otherwise \( \lim_{N_0 \to \infty} \int_{\Theta_3(N_0, x, z)} N_0 f_r(\theta)d\theta = \infty \), a contradiction due to (A6). Because \( \lim_{N_0 \to \infty} \Theta_1(N_0, x, z) = \emptyset \), taking the limit of (A6) as \( N_0 \to \infty \) yields

\[ 1 = \int_{\Theta} (1 + e^{\lim_{N_0 \to \infty} \mu^{agg}_r(N_0, x, z)/\epsilon} - e^{\mu(x)/\epsilon})^{-\epsilon}f_r(\theta)d\theta. \]

By Assumption 5, this equation implies that \( \lim_{N_0 \to \infty} \mu^{agg}_r(N_0, x, z) < \mu^{\max}_d \).

Part 2: Positivity of the disagreement effect

Because \( \mu^{\max}_d > \mu \), the effect of disagreement on prices is positive for \( e^{-\mu^{\max}_d x} < N_0 \leq 1 \). Because the effect decreases for \( 1 \leq N_0 \leq N_0^*(x, z) \), it is positive for \( N_0 > 1 \) if it is positive for \( N_0 \geq N_0^*(x, z) \) in the case that \( N_0^*(x, z) < \infty \), or if its asymptote as \( N_0 \to \infty \) is positive in the case that \( N_0^*(x, z) = \infty \).

In the first case, positivity obtains when \( \supp f_r \subset [-\mu/z, \mu^{\max}_d] \) and \( \int_{\Theta} f_r(\theta)d\theta = 0 \) because Lemma 3 showed that \( \mu^{agg}_r(N_0, x, z) > \mu \) for \( N_0 \geq N_0^*(x, z) \).

In the second case, by Proposition 2 and Jensen’s inequality, \( 1 > (e^{\mu^{\max}_d x/\epsilon} - e^{\mu(x)/\epsilon})^{-\epsilon} \), which implies that \( e^{\mu^{\max}_d x/\epsilon} > 1 + e^{\mu(x)/\epsilon} \). When \( N_0^*(x, z) = \infty \), the effect of disagreement on the house price asymptotes to \( (e^{\mu^{\max}_d x/\epsilon} - e^{\mu(x)/\epsilon} - 1)/(1 + e^{\mu(x)/\epsilon}) > 0 \).

Part 3: Marginal effect of small disagreement

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For any $N_0 < e^{-\mu_d x}$, we may find $z > 0$ small enough so that $N_0 < e^{-\mu_d^{\text{max}} x}$ because $\lim_{z \to 0} \mu_d^{\text{max}} = \bar{\mu}$. By the first part of Proposition 4, for such small $z$, $\partial p_0^b(N_0, x, z)/\partial z = 0$, proving the first part of formula.

For $e^{\mu_d x} \leq N_0 \leq 1$, we divide the formula in the first part of Proposition 4 by $z$ and take the limit as $z \to 0$ to obtain the expression in the second part of the formula.

Last, for each $N_0 > 1$, because $\lim_{z \to 0} N_0^*(x, z) = 1$ as shown in the proof of Proposition 2, for small enough $z > 0$ we have $N_0 \geq N_0^*(x, z)$. Because $\mu_d^{agg}(N_0, x, z) = \bar{\mu} + o(z)$ as shown by Lemma 3, $\partial \mu_d^{agg}(N_0, x, 0)/\partial z = 0$ and $\partial p_0^b(N_0, x, 0)/\partial z = 0$.

**Proposition 5**

As noted in the text, $p_0^b(N_0, 0, z) = p_0^b(N_0, 0, 0)$ because when $x = 0$, $\mu(\theta) = \bar{\mu}$ for all $\theta \in \Theta$, so $z$ becomes irrelevant for the equilibrium. As a result, we may take the formula for $p_0^b(N_0, 0, 0)$ given by Proposition 1 in the special case when $x = 0$ and use it for $p_0^b(N_0, 0, z)$. Combining this formula with that for $p_0^b(N_0, x, z)$ given by Proposition 4 yields the formulas in Proposition 5. The boom strictly increases for $e^{-\mu_d^{\text{max}} x} \leq N_0 \leq 1$ and strictly decreases for $1 \leq N_0 \leq N_0^*(x, z)$. Therefore, it is strictly maximized at $N_0 = 1$ as long as its value at $N_0 = 1$, which equals $(e^{\mu_d^{\text{max}} x}/e - 1)/2$, exceeds the boom for all $N_0 \geq N_0^*(x, z)$, which occurs as long as $\mu_d^{\text{max}} > \mu_d^{agg}(N_0, x, z)$ for all $N_0 \geq N_0^*(x, z)$. This inequality obtains by Assumption 5, as shown in the proof of Proposition 4.

**Proposition 6**

As shown in the proof of Lemma 2, in equilibrium the profit (utility for firm owners) of a developer equals $p_0^b L_0 + (p_1^b - p_0^b + k)L_1$ and the utility of potential residents equals $(v + p_1^b - p_0^b)H_0^{\text{buy}}$. The set of possible changes to the allocation are summarized by a cash transfer $\tau$ (which may vary across agents) with $\tau = 0$ and changes $\Delta L_1$ for each developer and $\Delta H_0^{\text{buy}}$ for each potential resident such that $\sum \Delta L_1 = \sum \Delta H_0^{\text{buy}}$. For a given realization of $p_1^b$, this change is a Pareto improvement only if $(p_1^b - p_0^b + k)\Delta L_1 + \tau \geq 0$ for all developers and $(v + p_1^b - p_0^b)\Delta H_0^{\text{buy}} + \tau \geq 0$ for all potential residents, with at least one strict inequality. Summing these inequalities across agents gives $\sum k\Delta L_1 + v\Delta H_0^{\text{buy}} > 0$.

We now show that the $z = 0$ equilibrium (described in the proof of Lemma 2) is Pareto efficient for any $p_1^b$. If $N_0 < 1$, a potential resident buys if and only if $v \geq p_0^b(N_0, x, z) - p_1^b(e^{\mu(\theta) x} N_0) = k$. The only feasible $\Delta H_0^{\text{buy}}$ are $-1$ for someone with $H_0^{\text{buy}} = 1$ and $1$ for someone with $H_0^{\text{buy}} = 0$. Because $\sum \Delta L_1 = \sum \Delta H_0^{\text{buy}}$, either one of these changes does not increase the welfare criterion given above. When $N_0 \geq 1$, $L_1 = 0$ for all developers and a potential resident buys only if $v \geq p_0^b(N_0, x, z) - p_1^b(e^{\mu(\theta) x} N_0) = k N_0^{1/e} \geq k$. The only feasible $\Delta L_1$ are positive, and the only feasible changes to $\Delta H_0^{\text{buy}}$ are $1$ for $v \leq k N_0^{1/e}$ and $-1$ for $v \geq k N_0^{1/e}$. The change to the welfare criterion above can never be positive resulting from these changes. As a result, the allocation under the $z = 0$ equilibrium is Pareto efficient for any $p_1^b$, meaning that it is belief-neutral Pareto efficient.

When $N_0 \leq e^{-\mu_d^{\text{max}} x}$, potential residents buy when $v \geq p_0^b(N_0, x, z) - p_1^b(e^{\mu(\theta) x} N_0)$. This difference is no greater than $k$ because $p_0^b(N_0, x, z) = 2k$ and $p_1^b(e^{\mu(\theta) x} N_0) \geq k$. Because all potential residents have $v \geq k$ by Assumption 1, buyers all have $v \geq k$, and all potential residents with $v > k$ buy. For the same argument given in the $z = 0$ equilibrium above, there does not exist a reallocation that improves welfare for each $p_1^b$, meaning that the equilibrium under the equilibrium with $z > 0$ is belief-neutral Pareto efficient when $N_0 \leq e^{-\mu_d^{\text{max}} x}$.

When $e^{-\mu_d^{\text{max}} x} < N_0 < N_0^*(x, z)$, $L_1 > 0$ for at least one developer. If $z > 0$, then by Assumption 4 there exists a positive measure of potential residents for whom $\theta < \theta_d^{\text{max}}$. These
potential residents buy only if \( v \geq p^h_0(N_0, x, z) - p^h_1(e^{\mu(\theta)x}N_0) > p^h_0(N_0, x, z) - p^h_1(e^{\mu_{\text{max}}x}N_0) = k, \)
where the latter equality uses an equilibrium condition from the proof of Lemma 2. It follows that there exists a positive measure of potential residents with \( v > k \) who do not buy. For a given \( p^h_1 \), we improve the allocation by setting \( \Delta L_1 = -1 \) and \( \tau = \tau^* \) for a developer holding land and setting \( \Delta H_{0 \text{buy}}^\text{buy} = 1 \) and \( \tau = -\tau^* \) for a potential resident with \( v > k \) who does not buy a house, where \( k + p^h_1 - p^h_0 \leq \tau^* \leq v + p^h_1 - p^h_0. \)

When \( N_0 \geq N_0^* (x, z) \), potential residents buy only if \( v \geq k(1+e^{\mu_{\text{agg}}x}N_0) - p^h_1(e^{\mu(\theta)x}N_0). \)
Due to Assumption 4, \( z > 0 \) implies that \( \mu(\theta) \) varies across potential residents. Because \( N_0 > 1 \) and \( p^h_1(e^{\mu(\theta)x}N_0) \) varies across potential residents. It follows that the purchase cutoff varies across potential residents, meaning that we can find a potential resident with \( H_{0 \text{buy}}^\text{buy} = 1 \) and \( v = v^1 \) and a potential resident with \( H_{0 \text{buy}}^\text{buy} = 0 \) and \( v = v^2 \) with \( v^1 < v^2 \). Setting \( \Delta H_{0 \text{buy}}^\text{buy} = -1 \) and \( \tau = \tau^* \) for the first potential resident and \( \Delta H_{0 \text{buy}}^\text{buy} = 1 \) and \( \tau = -\tau^* \) for the second potential resident strictly increases the welfare objective if \( v^1 + p^h_1 - p^h_0 \leq \tau^* \leq v^2 + p^h_1 - p^h_0. \)
Developers who can access the equity market choose a share $\alpha^{sell} \in [0, 1]$ of the claim to their total $t = 1$ liquidation value to sell at $t = 0$. The price of this claim equals $p_0^\pi$, which may vary across developers. Each of these developers may also pay itself a dividend $\delta$ at $t = 0$ using its available cash flow. Finally, land that remains undeveloped at the end of $t = 0$ pays a dividend $k_1 > 0$ at $t = 1$; we focus on the limiting equilibria as $k_1 \rightarrow 0.1$ The optimal behavior for such a developer is to choose $\delta^*, (\alpha^{sell})^*, (H_0^{sell})^*, (L_0^{buy})^*$, and $(H_0^{build})^*$ from

$$\arg\max_{\delta, \alpha^{sell}, H_0^{sell}, L_0^{buy}, H_0^{build}} \delta + (1 - \alpha^{sell})E\pi(p_1^h, p_1^l, H_1, L_1, B_1)$$

subject to

$$\alpha^{sell} \in [0, 1]$$

$$H_0^{sell} \leq H_0^{build}$$

$$H_0^{build} \leq L_0 + L_0^{buy}$$

$$H_1 = H_0^{build} - H_0^{sell}$$

$$L_1 = L_0 + L_0^{buy} - H_0^{build}$$

$$B_1 = p_0^h H_0^{sell} - p_0^l L_0^{buy} - 2k H_0^{build} + \alpha^{sell} p_0^\pi - \delta$$

$$0 \leq B_1$$

$$0 \leq \delta.$$

Developers who cannot access the equity market face the same problem with the additional constraint $\alpha^{sell} = 0$. For all developers, the $t = 1$ problem remains the same as before.

A unit measure of equity investors chooses a share $\alpha^{buy}$ of the claim to each developer’s $t = 1$ liquidation value to buy at $t = 0$. The chosen $\alpha^{buy}$ may differ for each investor-developer pair. Each investor faces a proportional cost $k_s \in (0, 1)$ for each dollar invested in a negative position, and the most negative position that can be taken is $-\overline{\alpha}$, where $\overline{\alpha} > 0$. For a given developer, an equity investor chooses $(\alpha^{buy})^*$ from

$$\arg\max_{\alpha^{buy}} \alpha^{buy}E\pi(p_1^h, p_1^l, H_1, L_1, B_1) - \max(\alpha^{buy}, (1 - k_s)\alpha^{buy})p_0^\pi$$

subject to

$$-\overline{\alpha} \leq \alpha^{buy}$$

$$H_1 = (H_0^{build})^* - (H_0^{sell})^*$$

$$L_1 = L_0 + (L_0^{buy})^* - (H_0^{build})^*$$

$$B_1 = p_0^h (H_0^{sell})^* - p_0^l (L_0^{buy})^* - 2k (H_0^{build})^* + (\alpha^{sell})^* p_0^\pi - \delta^*,$$

where $E$ denotes the equity investor’s expectation and $\delta^*, (\alpha^{sell})^*, (H_0^{sell})^*, (L_0^{buy})^*$, and $(H_0^{build})^*$ denote the actions chosen by the developer.

The potential resident problems remain the same. Prices $p_0^h$, $p_0^l$, and $p_0^\pi$ constitute an equilibrium when, in addition to the clearing of land and housing markets described in Section 1, the

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1This dividend leads to a positive land price at $t = 0$ that guarantees the existence of equilibrium when $E p_1^l = 0$ for all equity investors but $E p_1^l > 0$ for some developers. The proof of Proposition 7 further discusses this issue.
following holds: for each developer, \((\alpha^{sell})^*\) equals the sum across equity investors of \((\alpha^{buy})^*\).

We now characterize equilibrium. The first lemma simplifies the objective of each developer:

**Lemma B1.** In equilibrium, each developer chooses \(\alpha^{sell}\) and \(L_1 \geq 0\) such that \(p_0^B(L_0 - L_1) + \alpha^{sell}p_0^\pi \geq 0\) to maximize \(p_0^B(L_0 - L_1) + \alpha^{sell}p_0^\pi + (1 - \alpha^{sell})E(p_1^k + k_l)L_1\).

**Proof.** In all of the \(t = 1\) equilibria characterized in the proof of Lemma 1, \(p = p_1^B H_1 + (p_1^I + k_l)L_1 + B_1\) (\(p_1^I\) is the ex-dividend price). At \(t = 0\), the developer maximizes \(\delta + (1 - \alpha^{sell})E(p_1^B H_1 + (p_1^I + k_l)L_1 + B_1)\). From substituting the \(H_1\) and \(L_1\) constraints into the \(B_1\) constraint, we have \(B_1 = -p_0^B H_1 + p_0^B(L_0 - L_1) + (p_0^I - p_0^L - 2k)H_0^{build} + \alpha^{sell}p_0^\pi - \delta\). In equilibrium \(p_0^B = p_0^B + 2k\), for otherwise each developer would want to build a positively or negatively infinite amount of housing. Therefore \(B_1 = -p_0^B H_1 + p_0^B(L_0 - L_1) + \alpha^{sell}p_0^\pi - \delta\). The developer maximizes \(\delta + (1 - \alpha^{sell})E((p_1^B - p_0^B)H_1 + (p_1^I + k_l - p_0^L)L_1 + p_0^L + \alpha^{sell}p_0^\pi - \delta)\) by choosing \(H_1, L_1 \geq 0\), \(\alpha^{sell} \in [0, 1]\), and \(\delta\) such that \(B_1 \geq 0\). Because \(p_1^B - p_0^B = p_1^I - p_0^L - k < p_1^I + k_l - p_0^L\), in equilibrium all developers set \(H_1 = 0\) (if \(H_1 > 0\) is optimal, then the developer wants an infinite \(L_1\)). The objective weakly increases in \(\delta\) for \(\alpha^{sell} \in [0, 1]\), so it is maximized at \(\delta = -p_0^B H_1 + p_0^B(L_0 - L_1) + \alpha^{sell}p_0^\pi\), the largest possible value given the \(B_1 \geq 0\) constraint. The \(\delta \geq 0\) constraint produces \(p_0^B(L_0 - L_1) + \alpha^{sell}p_0^\pi \geq 0\). The objective simplifies to \(p_0^B(L_0 - L_1) + \alpha^{sell}p_0^\pi + (1 - \alpha^{sell})E(p_1^k + k_l)L_1\), as claimed. \(\square\)

The developer objective consists of three terms: profits from current land sales, revenues from equity offerings, and profits expected at \(t = 1\) from end-of-period land holdings. The next lemma delivers the equilibrium price of equity:

**Lemma B2.** In equilibrium, \(p_0^\pi = (p_1^B(e^{\mu_{max}x}N_0) + k_l)L_1\) for any developer for whom \((\alpha^{sell})^* > 0\).

**Proof.** As shown in the proof of Lemma B1, each developer sets \(H_1 = 0\) and sets \(B_1 = 0\) when \(\alpha^{sell} > 0\). The liquidation value of the developer becomes \(\pi = (p_1^I + k_l)L_1\). If \(p_0^\pi < (p_1^I(e^{\mu_{max}x}N_0) + k_l)L_1\), then the equity investors for whom \(\theta = \alpha_i^{buy}\) want to set \(\alpha^{buy}\) arbitrarily large. The equity market cannot clear in this case because the maximal aggregate short position across equity investors is bounded at \(-\pi\). Therefore \(p_0^\pi \geq (p_1^I(e^{\mu_{max}x}N_0) + k_l)L_1\). If this inequality is strict, then \((\alpha^{buy})^* \leq 0\) for all equity investors, preventing clearing in the equity market. The only equilibrium outcome is the one given in the lemma. \(\square\)

The price of any traded claim equals the most optimistic equity investor valuation of the land held by that developer at the end of \(t = 0\). In this sense, traded developers act like land hedge funds by raising equity against speculative land investments. To make this point clear, the following lemma relates the equilibrium prices of developer equity and the land they hold:

**Lemma B3.** In equilibrium, \(p_0^\pi = p_0^B L_1\) for any developer for whom \((\alpha^{sell})^* > 0\).

**Proof.** We prove this claim by delineating all possible choices by developers in equilibrium. By substituting Lemma B2 into Lemma B1, we rewrite the developer problem as choosing

\[
L_1^*, (\alpha^{sell})^* \in \arg \max_{L_1, \alpha^{sell}} \quad \begin{align*}
p_1^B L_0 + \left(\alpha^{sell} p_1^B(e^{\mu_{max}x}N_0) + (1 - \alpha^{sell})p_1^I(e^{\mu x}N_0) + k_l - p_0^L\right) L_1 \\
\text{subject to} \quad \begin{align*}
p_0^B L_1 &\leq p_0^B L_0 + \alpha^{sell}(p_1^I(e^{\mu_{max}x}N_0) + k_l)L_1 \\
0 &\leq L_1 \\
\alpha^{sell} &\in [0, 1] \ (\text{with access to equity market}) \\
\alpha^{sell} &\equiv 0 \ (\text{without access to equity market})
\end{align*}
\]
A developer that cannot access the equity market sets \((\alpha^{sell})^* = 0\) and chooses

\[
L_1^* = L_0 \quad \text{if } p_0^l < p_1^l(e^{\mu_i}x N_0) + k_l
\]

\[
L_1^* \in [0, L_0] \quad \text{if } p_0^l = p_1^l(e^{\mu_i}x N_0) + k_l
\]

\[
L_1^* = 0 \quad \text{if } p_0^l > p_1^l(e^{\mu_i}x N_0) + k_l
\]

if \(p_0^l > 0\). If \(p_0^l \leq 0\), then \(L_1^*\) does not exist because the developer always increases its objective function without violating the constraints by increasing \(L_1\) beyond \(L_0\). Similarly, if \(p_0^l < p_1^l(e^{\mu_i}x N_0) + k_l\) then \(L_1^*\) does not exist for developers with access to the equity market. With \(\alpha^{sell} = 1\), increasing \(L_1\) always increases the objective function while obeying the constraints. If \(p_0^l = p_1^l(e^{\mu_i}x N_0) + k_l\), then the optimal choices for developers with access to the equity market are

\[
L_1^* = \frac{L_0}{1 - (\alpha^{sell})^*} \quad \text{and } (\alpha^{sell})^* \in [0, 1) \quad \text{if } p_1^l(e^{\mu_i}x N_0) < p_1^l(e^{\mu_i}x N_0)
\]

\[
L_1^* \geq 0 \quad \text{and } (\alpha^{sell})^* = 1
\]

or

\[
L_1^* \in \left[ \frac{L_0}{1 - (\alpha^{sell})^*} \right] \quad \text{and } (\alpha^{sell})^* \in [0, 1) \quad \text{if } p_1^l(e^{\mu_i}x N_0) = p_1^l(e^{\mu_i}x N_0)
\]

\[
L_1^* = 0 \quad \text{and } (\alpha^{sell})^* \in [0, 1) \quad \text{if } p_1^l(e^{\mu_i}x N_0) > p_1^l(e^{\mu_i}x N_0).
\]

The first case follows because if \(\alpha^{sell} < 1\), the objective strictly increases in \(L_1\) and so is maximized at \(L_1^* = L_0/(1 - \alpha^{sell})\) with a value of \((p_1^l(e^{\mu_i}x N_0) + k_l)L_0\). This value exceeds \(p_0^lL_0\), the objective function value obtained when \(\alpha^{sell} = 1\). In the second case of the optimal developer choices, the objective is independent of \(L_1\) and \(\alpha^{sell}\), so the developer may choose any feasible combination. In the third case, the objective decreases in \(L_1\) if \(\alpha^{sell} < 1\), leading to \(L_1^* = 0\); if \(\alpha^{sell} = 1\), then the objective is independent of \(L_1\), permitting the developer to choose any feasible value for \(L_1^*\). Finally, if \(p_0^l > p_1^l(e^{\mu_i}x N_0) + k_l\) then

\[
L_1^* = L_0 \quad \text{and } (\alpha^{sell})^* = 0 \quad \text{if } p_0^l < p_1^l(e^{\mu_i}x N_0) + k_l
\]

\[
L_1^* \in [0, L_0] \quad \text{and } (\alpha^{sell})^* = 0
\]

or

\[
L_1^* = 0 \quad \text{and } (\alpha^{sell})^* \in [0, 1] \quad \text{if } p_0^l = p_1^l(e^{\mu_i}x N_0) + k_l
\]

\[ L_1^* = 0 \quad \text{and } (\alpha^{sell})^* \in [0, 1] \quad \text{if } p_0^l > p_1^l(e^{\mu_i}x N_0) + k_l \]

are the optimal choices for developers with access to the equity market. In the first case, the value of the objective function at the given choices equals \((p_1^l(e^{\mu_i}x N_0) + k_l)L_0\). For \(\alpha^{sell} \geq (p_1^l(e^{\mu_i}x N_0) + k_l - p_0^l)/p_1^l(e^{\mu_i}x N_0) - p_1^l(e^{\mu_i}x N_0))\), the coefficient in the objective function on \(L_1\) is non-positive, meaning that it is maximized at \(L_1^* = 0\) with a value of \(p_0^lL_0\), which is less than the maximized value when \(L_1^* = L_0\) and \((\alpha^{sell})^* = 0\). For \(\alpha^{sell} \in (0, (p_1^l(e^{\mu_i}x N_0) + k_l - p_0^l)/p_1^l(e^{\mu_i}x N_0) - p_1^l(e^{\mu_i}x N_0))]\), the value of the objective function is maximized at \(L_1^* = L_0\) and \((\alpha^{sell})^* = 0\).
\( k_1 - p_0^l \left( p_I^h(e^{\mu'x} N_0) - p_I^l(e^{\mu^{max}x} N_0) \right) \), the coefficient on \( L_1 \) in the objective function is positive, meaning that it is maximized at \( L_1 = p_0^l L_0 / \left( p_0^l - \alpha^{sell}(p_I^l(e^{\mu^{max}x} N_0) + k_l) \right) \) with a value of \( p_0^l L_0 (1 - \alpha^{sell})(p_I^l(e^{\mu'x} N_0) + k_l) / \left( p_0^l - \alpha^{sell}(p_I^l(e^{\mu^{max}x} N_0) + k_l) \right) \). This value is less than the maximized value when \((\alpha^{sell})^* = 0\) because for such \(\alpha^{sell}\), \((1 - \alpha^{sell})p_0^l < p_0^l - \alpha^{sell}(p_I^l(e^{\mu^{max}x} N_0) + k_l)\). We have proved that the given choices are optimal in the first case. The proof that the choices are optimal in the second case is similar. The maximized objective equals \( p_0^l L_0 \). If \( \alpha^{sell} > 0 \), then the coefficient on \( L_1 \) in the objective is negative, leading to \( L_1^* = 0 \). If \( \alpha^{sell} = 0 \), then the coefficient on \( L_1 \) in the objective is 0, leading to any feasible choice of \( L_1^* \). Finally, in the third case, the coefficient on \( L_1 \) in the objective is negative for all \( \alpha^{sell} \), leading to \( L_1^* = 0 \), in which case \((\alpha^{sell})^* \) does not affect the objective.

In all of the equilibrium choices we have just listed, \((\alpha^{sell})^* > 0\) only if \( L_1^* = 0 \) or if \( p_0^l = p_I^l(e^{\mu^{max}x} N_0) + k_l \). In either case, \( p_0^l = p_0^l L_1 \) by Lemma B2.

We now use Lemmas B1 and B2 to formulate and prove a lemma that characterizes the equilibrium house price at \( t = 0 \) as \( k_l \to 0 \). The lemma relies on the following definitions: \( \mu_i^{max} = \mu(\theta, \max) \) is the belief of the most optimistic equity investor, \( \theta_d^{sup} = \sup\{\theta \in \text{supp} f_d | L_0 > 0\} \) is the least upper-bound of the beliefs of developers endowed with land, and \( N_0^* (x, z, f_r, \theta_i^{max}) \) is the value of \( N_0^* (x, z) \) in Proposition 2 given \( f_r \) and \( \theta_i^{max} \).

**Lemma B4.** Suppose that \( x, z > 0 \). If \( \sum_{\theta > \theta_i^{max}} L_0 = 0 \), then the limit of the equilibrium house price at \( t = 0 \) as \( k_l \to 0 \) equals

\[
 p_0^h(N_0, x, z) = \begin{cases} 
 2k & \text{if } N_0 \leq e^{-\mu_i^{max}x} \\
 k + ke^{\mu_i^{max}x / \epsilon} N_0^{1/\epsilon} & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^* (x, z, f_r, \theta_i^{max}) \\
 k(1 + e^{\mu_d^{agg}(N_0, x, z) / \epsilon}) N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^* (x, z, f_r, \theta_i^{max}). 
\end{cases}
\]

If \( \sum_{\theta \geq \theta_i^{max}} L_0 > 0 \), then the limit of the equilibrium house price at \( t = 0 \) as \( k_l \to 0 \) equals

\[
 p_0^h(N_0, x, z) = \begin{cases} 
 2k & \text{if } N_0 \leq \min (e^{-\mu_i^{max}x}, N_0^* (x, z)) \\
 k + ke^{\mu_i^{max}x / \epsilon} N_0^{1/\epsilon} & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^* (x, z) \\
 k + ke^{\mu_d^{agg}(N_0, x, z) / \epsilon} N_0^{1/\epsilon} & \text{if } N_0^* (x, z) < N_0 < N_0^* (x, z, f_r, \theta_d^{sup}) \\
 k(1 + e^{\mu_d^{agg}(N_0, x, z) / \epsilon}) N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^* (x, z, f_r, \theta_d^{sup}). 
\end{cases}
\]

Here \( \mu_d^{agg}(N_0, x, z) \) increases in \( N_0 \) and depends on the beliefs and endowments of only those developers for whom \( \theta > \theta_i^{max} \) and \( L_0 > 0 \), and \( N_0^* (x, z, f_r, \theta_d^{sup}) \geq N_0^{**} (x, z) \in \mathbb{R} \cup \{\infty\} \) with equality if and only if \( N_0^{**} (x, z) = \infty \), which occurs if and only if \( \int_{\theta \geq \theta_i^{max}} f_r(\theta)d\theta = 0 \) and \( \int_{\theta < \theta_i^{max}} (e^{\mu_i^{max}x / \epsilon} - e^{\mu(\theta)x / \epsilon}) - \epsilon f_r(\theta)d\theta \leq \sum_{\theta \leq \theta_i^{max}} L_0 / S \).

**Proof of Lemma B4.** The proof of Lemma B3 fully characterized developer choices of end-of-period landholdings at \( t = 0 \) given \( p_0^l \). The land price constitutes an equilibrium when the space demanded by potential residents given \( p_0^l \) plus the sum of \( L_1 \) across developers equals \( S \) (the proof of Lemma B1 shows that \( H_1 = 0 \) for all developers). If \( p_0^l = p_I^l(e^{\mu_i^{max}x} N_0) + k_l \), then the total \( L_1 \) across developers can take on any value at least \( \sum_{\theta} p_I^l(e^{\mu_i^{max}x} N_0) < p_I^l(e^{\mu(\theta)x} N_0) \). Equilibrium holds in this case if and only if

\[
 \sum_{\theta} p_I^l(e^{\mu_i^{max}x} N_0) \geq \int_{\theta} N_0 D(p_I^h(e^{\mu_i^{max}x} N_0) + k + k_l - p_I^l(e^{\mu(\theta)x} N_0)) f_r(\theta)d\theta. \]  

(B1)
By the same argument about the right side of (A1) in the proof of Proposition 2, the right side of (B1) weakly and continuously increases in $N_0$ and $\to 0$ as $N_0 \to 0$. The left side of (B1) equals

$$\sum_{\theta | |p^h_i(e^{\mu(x \mid z \mid N_0)} N_0) \geq p^h_i(e^{\mu(\theta \mid z \mid N_0)})} L_0/S = \begin{cases} 1 & \text{if } N_0 \leq e^{-\mu_{d_{\text{max}}}^x} \\ 1 - \sum_{\theta \epsilon(\theta \mid z \mid N_0) N_0 > 1} L_0/S & \text{if } e^{-\mu_{d_{\text{max}}}^x} \leq N_0 \leq e^{-\mu_{i_{\text{max}}}^x} \\ 1 - \sum_{\theta > \mu_{i_{\text{max}}}^x} L_0/S & \text{if } N_0 \geq e^{-\mu_{i_{\text{max}}}^x}, \end{cases}$$

which weakly decreases in $N_0$ and is left-continuous. As a result, there is $N_0^{x^*}(x, z, k, i) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ such that (B1) holds if and only if $N_0 \leq N_0^{x^*}(x, z, k, i)$. Because the right side of (B1) decreases in $k_i$, $N_0^{x^*}(x, z, k, i)$ increases in $k_i$, meaning that $N_0^{x^*}(x, z, f, \theta_{i_{\text{max}}}^x, k_i) = \lim \max_{k_i \to 0} N_0^{x^*}(x, z, f, \theta_{i_{\text{max}}}^x, k_i)$ exists.

We pause here to prove two needed facts about $N_0^{x^*}(x, z, k_i)$. As a point of notation, define $N_0^{x^*}(x, z, f, \theta_{d_{\text{max}}}^x, k_i)$ to be the value of $N_0^x(x, z)$ obtained from (A1) with $k + k_i$ in place of $k$ inside the integral. First: if $\sum_{\theta \epsilon(\theta \mid z \mid N_0) N_0} L_0 = 0$, then the left side of (B1) reduces to 1. It follows from comparison with (A1) that $N_0^{x^*}(x, z, f, \theta_{i_{\text{max}}}^x, k_i)$ and $N_0^{x^*}(x, z) = N_0^x(x, z, f, \theta_{i_{\text{max}}}^x, k_i)$ in this case. Second: by the same argument used in Proposition 2 to analyze (A1), the limit of the right side of (B1) as $N_0 \to \infty$ equals $\infty$ if $\int_{\theta \epsilon(\theta \mid z \mid N_0) N_0} f_r(\theta) d\theta \geq 0$ and equals $\int_{\theta > \mu_{i_{\text{max}}}^x} e^{\mu(\theta \mid z \mid N_0) N_0} f_r(\theta) d\theta$ otherwise. It follows that $N_0^{x^*}(x, z) = \infty$ if and only if the conditions given in Lemma B4 hold.

In the second possible equilibrium, $p_0^h > p_1^h(e^{\mu_{d_{\text{max}}}^x N_0}) + k_i$. In this case, the total $L_1$ across developers may take on any value between $\sum_{\theta \epsilon(\theta \mid z \mid N_0) N_0 } L_0 = 0$ and $\sum_{\theta \epsilon(\theta \mid z \mid N_0) N_0} L_0$. Equilibrium holds if potential residents demand for space at $p_0^h = p_0^h + 2k$ equals the remaining land not held by developers; that is, if $p_0^h$ satisfies

$$\sum_{\theta \epsilon(\theta \mid z \mid N_0) N_0} L_0/S \leq \int_{\Theta} N_0^D(p_0^h - p_1^h(e^{\mu(\theta \mid z \mid N_0)}) f_r(\theta) d\theta \leq \sum_{\theta \epsilon(\theta \mid z \mid N_0) N_0} L_0/S. \tag{B2}$$

Such a $p_0^h$ exists and if and only if (B1) fails. Indeed, suppose (B1) holds. The left side of (B2) weakly increases in $p_0^h$, while the middle strictly decreases for $p_0^h \geq p_1^h(e^{\mu_{d_{\text{max}}}^x N_0}) + k + k_i$ because $\theta < 0 < \theta_{i_{\text{max}}}^x$ for a positive measure of potential residents (Assumption 4). If (B1) holds, then the left side of (B2) is at least the middle when $p_0^h = p_1^h(e^{\mu_{d_{\text{max}}}^x N_0}) + k + k_i$, meaning that for larger $p_0^h$, the left strictly exceeds the middle in violation of (B2). Now suppose that (B1) fails. Then the middle of (B2) exceeds the right side at $p_0^h = p_1^h(e^{\mu_{d_{\text{max}}}^x N_0}) + k + k_i$. Because the middle strictly and continuously decreases to 0 with $p_0^h \geq p_1^h(e^{\mu_{d_{\text{max}}}^x N_0}) + k + k_i$, there exists a unique solution to (B2), which we deem $p_0^h(N_0, x, z, f, \theta_{i_{\text{max}}}^x, k_i)$. Existence and uniqueness follow from the fact that the greatest lower bound of the $p_0^h$ for which the left inequality fails equals the lowest upper bound of the $p_0^h$ for which the right inequality fails.

We further partition this possible equilibrium into two cases. Set $\mu_{\text{sup}}^d = \mu(\theta_{d_{\text{sup}}}^x)$. In the first case,

$$1 \geq \int_{\Theta} N_0^D(p_0^h(e^{\mu_{d_{\text{sup}}}^x N_0}) + k + k_i - p_1^h(e^{\mu(\theta \mid z \mid N_0)}) f_r(\theta) d\theta. \tag{B3}$$

At $p_0^h = p_1^h(e^{\mu_{d_{\text{sup}}}^x N_0}) + k + k_i$, the right side of (B2) equals 1. As a result, if (B3) fails, then $p_0^h(N_0, x, z, k_i)$ satisfies (A2). If (B3) holds, then if $p_0^h > p_1^h(e^{\mu_{d_{\text{sup}}}^x N_0}) + k + k_i$, the left and right of (B2) equal 1 while the middle is less than 1. As a result, $p_0^h(N_0, x, z, k_i) \leq p_1^h(e^{\mu_{d_{\text{sup}}}^x N_0}) + k + k_i$. By the same argument given in the proof of Proposition 2 concerning (A1), (B3) holds if and only if $N_0 \leq N_0^x(x, z, f, \theta_{i_{\text{max}}}^x, k_i)$.

In summary, a unique equilibrium house price at $t = 0$ exists. If $N_0 \leq N_0^{x^*}(x, z, k_i)$, then we have $p_0^h(N_0, x, z, k_i) = p_1^h(e^{\mu_{d_{\text{max}}}^x N_0}) + k + k_i$. If $N_0^{x^*}(x, z, k_i) < N_0 < N_0^{x^*}(x, z, f, \theta_{d_{\text{sup}}}^x, k_i)$,
then \( p_{\theta}^{\epsilon}(e_{\epsilon}^{\max}N_0) + k + k_l < p_{\theta}^{\epsilon}(N_0, x, z, k_l) \leq p_{\theta}^{\epsilon}(e_{\epsilon}^{\sup}N_0) + k + k_l \). If \( N_0 > N_0^*(x, z, k_l) \) and \( N_0 \geq N_0^*(x, z, f_r, \theta_{\epsilon}^{\sup}) \), then \( p_{\theta}^{\epsilon}(N_0, x, z, k_l) \) satisfies (A2).

If \( \sum_{\theta > \theta_i^{\max}} L_0 = 0 \), then \( \mu_{\epsilon}^{\sup} = \mu_{\epsilon}^{\max} \). In this case, \( N_0^*(x, z, k_l) = N_0^*(x, z, f_r, \theta_{\epsilon}^{\sup}) \), where the equality was proved earlier and the inequality follows because \( N_0^* \) increases in its fourth argument. As a result, the equilibrium house price in this case equals

\[
p_{\theta}^{\epsilon}(N_0, x, z, k_l) = \begin{cases} 2k + k_l & \text{if } N_0 \leq e^{-\mu_{\epsilon}^{\max}} \\ k + k_l & \text{if } e^{-\mu_{\epsilon}^{\max}} < N_0 < N_0^*(x, z, f_r, \theta_{\epsilon}^{\sup}) \\ (k + 1 + e^{h_{\epsilon}^{agg}(N_0, x, z)/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z, f_r, \theta_{\epsilon}^{\max}, k_l). \end{cases}
\]

Taking the limit as \( k_l \to 0 \) yields the formula in Lemma B4.

If \( \sum_{\theta > \theta_i^{\max}} L_0 > 0 \), then \( \mu_{\epsilon}^{\sup} > \mu_{\epsilon}^{\max} \). From comparing (B1) to (B3), we see that \( N_0^*(x, z, k_l) \leq N_0^*(x, z, f_r, \theta_{\epsilon}^{\sup}, k_l) \) and \( N_0^*(x, z, \theta_{\epsilon}^{\max}) \leq N_0^*(x, z, f_r, \theta_{\epsilon}^{\sup}) \), with equality in each if and only if the respective left side equals \( \infty \). If \( N_0^*(x, z, k_l) < N_0 \leq N_0^*(x, z, f_r, \theta_{\epsilon}^{\sup}, k_l) \), then \( p_{\theta}^{\epsilon}(e_{\epsilon}^{\max}N_0) + k + k_l < p_{\theta}^{\epsilon}(N_0, x, z, k_l) \leq p_{\theta}^{\epsilon}(e_{\epsilon}^{\sup}N_0) + k + k_l \). Over this range, the only developers on which (B2) depends are those with positive land holdings and beliefs in \( \{\theta | p_{\theta}^{\epsilon}(e_{\epsilon}^{\max}N_0) < p_{\theta}^{\epsilon}(e_{\theta_{\epsilon}^{\max}}xN_0)\} = \{\theta | \theta > \theta_{\epsilon}^{\max}\} \). It follows that \( p_{\theta}^{\epsilon}(N_0, x, z, k_l) \) in this range depends on only these developers. Because \( p_{\theta}^{\epsilon}(e_{\epsilon}^{\sup}N_0) = ke_{\epsilon}^{\sup}N_0^{1/\epsilon} \) in this range, there exists a unique \( \mu_{\epsilon}^{agg}(N_0, x, z, k_l) \in [-\log(N_0)/x, \mu_{\epsilon}^{sup}] \) such that on this range of \( N_0, p_{\theta}^{\epsilon}(N_0, x, z, k_l) = k + ke_{\epsilon}^{agg}(N_0, x, z, k_l)x/\epsilon N_0^{1/\epsilon} + k_l \).

Because \( p_{\theta}^{\epsilon}(N_0, x, z, k_l) \) increases in \( k_l \) and is bounded on this range, \( \lim_{k_l \to 0} \mu_{\epsilon}^{agg}(N_0, x, z, k_l) \) exists; we deem it \( \mu_{\epsilon}^{agg}(N_0, x, z) \). The middle of (B2) increases in \( N_0 \), as shown in the the proof of Proposition 2, so \( \mu_{\epsilon}^{agg}(N_0, x, z) \) increases in \( N_0 \). Putting everything together, we have that

\[
p_{\theta}^{\epsilon}(N_0, x, z, k_l) = \begin{cases} 2k + k_l & \text{if } N_0 \leq \min(e^{-\mu_{\epsilon}^{max}}, N_0) \\ k + k_l & \text{if } e^{-\mu_{\epsilon}^{max}} < N_0 < N_0^*(x, z, k_l) \\ k + k_{\mu_{\epsilon}^{agg}(N_0, x, z, k_l)x/\epsilon N_0^{1/\epsilon}} + k_l & \text{if } N_0^*(x, z, k_l) < N_0 < N_0^*(x, z, f_r, \theta_{\epsilon}^{sup}, k_l) \\ (k + 1 + e^{h_{\epsilon}^{agg}(N_0, x, z)/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z, f_r, \theta_{\epsilon}^{\max}, k_l). \end{cases}
\]

when \( \sum_{\theta > \theta_i^{\max}} L_0 > 0 \). Taking the limit as \( k_l \to 0 \) yields the formula in Lemma B4.

The only point at which \( k_l > 0 \) was used is for the existence of equilibrium when \( p_{\theta}^{\epsilon}(N_0, x, z, k_l) = 2k + k_l \). In this case, \( p_{\theta}^{\epsilon}(N_0, x, z, k_l) = k_l \), but we showed earlier that \( p_{\theta}^{\epsilon} = 0 \) never can be an equilibrium. This equilibrium exists only as a limit as \( k_l \to 0 \).

The case when \( \sum_{\theta > \theta_i^{\max}} L_0 > 0 \) and \( N_0^*(x, z) < N_0 < N_0^*(x, z, f_r, \theta_{\epsilon}^{sup}) \) deserves some explanation, as the equilibrium house price in this region looks quite different than any of the prices in Proposition 2. This case occurs when demand from potential residents is at least equal to the space held by developers for whom \( \theta \leq \theta_i^{\max} \), but is not as large as the entire space \( S \). In such an equilibrium, developers for whom \( \theta > \theta_i^{\max} \) become the marginal owners of space and hold some land in equilibrium. The equilibrium house price aggregates the beliefs of such landowning developers through \( \mu_{\epsilon}^{agg} \). This case always occurs unless demand from potential residents when the optimistic equity investors price space is never large enough to cut into the landholdings of these very optimistic developers; this condition is precisely the one at the end of Lemma B4.

Finally, we build on the proof of Lemma B4 to prove Proposition 7.

**Proof of Proposition 7.** The claim that the equilibrium house price equals \( p_{\theta}^{\epsilon}(N_0, x, z, f_r, f_l) \) when \( \sum_{\theta > \theta_i^{\max}} L_0 = 0 \) follows immediately from comparing the pricing formula in Lemma B4 to that in Proposition 2.
To prove the remaining claims, we first solve for the optimal equity purchases for investors. By Lemma B2, the objective function for an equity investor with respect to a given developer is to maximize $\alpha^{buy}(p_1^i(e^{\mu(\theta)x}N_0) + k_l)L_1 - \max(\alpha^{buy}, (1 - k_s)\alpha^{buy})(p_1(e^{\mu^{max}x}N_0) + k_l)L_1$ subject to $\alpha^{buy} \geq -\bar{\alpha}$. If $L_1 > 0$, then the optimal choice for the equity investor is

\[
\begin{align*}
(\alpha^{buy})^* & \geq 0 & & \text{if } p_1^i(e^{\mu(\theta)x}N_0) = p_1^i(e^{\mu^{max}x}N_0) \\
(\alpha^{buy})^* & = 0 & & \text{if } p_1^i(e^{\mu(\theta)x}N_0) \in \left((1 - k_s)p_1(e^{\mu^{max}x}N_0) - k_sk_l, p_1(e^{\mu^{max}x}N_0)\right) \\
(\alpha^{buy})^* & \in [-\bar{\alpha}, 0] & & \text{if } p_1^i(e^{\mu(\theta)x}N_0) = (1 - k_s)p_1^i(e^{\mu^{max}x}N_0) - k_sk_l \\
(\alpha^{buy})^* & = -\bar{\alpha} & & \text{if } p_1^i(e^{\mu(\theta)x}N_0) < (1 - k_s)p_1^i(e^{\mu^{max}x}N_0) - k_sk_l.
\end{align*}
\]

When $xz = 0$, $p_1^i(e^{\mu(\theta)x}N_0) = p_1^i(e^{\mu^{max}x}N_0) > (1 - k_s)p_1^i(e^{\mu^{max}x}N_0) - k_sk_l$ because $k_s > 0$, so $(\alpha^{buy})^* \geq 0$ for all equity investors and developers for whom $L_1 > 0$. The claim that the aggregate value of short claims equals zero when $xz = 0$ is proved. For the second claim about the $xz = 0$ case, first consider the possibility that $p_0^s > p_1^i(e^{\mu^{max}x}N_0) + k_l$. Then the proof of Lemma B4 shows that $L_1^* = 0$ for all developers and that $(\alpha^{sell})^* = 0$ is possible for all developers, meaning that an equilibrium exists in which no equity issuance occurs and in which $(H_0^{build})^* = L_0$ and $(L_0^{buy})^* = 0$ for all developers. Now consider the other possibility, that $p_0^s = p_1^i(e^{\mu^{max}x}N_0) + k_l$. Then by the proof of Lemma B4, each developer may choose $(\alpha^{sell})^* = 0$ and $L_1^* \leq L_0$. As a result, no equity is issued, and the sum of $L_1^*$ across developers can take on any value between 0 and $S$, meaning that we may find an equilibrium in which $(L_0^{buy})^* = 0$ for all developers and $(H_0^{build})^*$ is chosen to clear the housing market.

We turn now to the remaining claims about the $xz > 0$ case. We define $N_{0^{***}}(x, z, k_l)$ to be the least upper bound of $N_0$ such that

\[
\sum_{\theta < \theta_{\max}} L_0/S > \int_{\Theta} N_0 D(p_1^h(e^{\mu^{max}x}N_0) + k_l - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta) d\theta. \tag{B4}
\]

As discussed in the proof of Lemma B4, the right side of (B4) continuously increases in $N_0$ and limits to 0 as $N_0 \to 0$, so $N_{0^{***}}(x, z, k_l) \in \mathbb{R}_{>0} \cup \{\infty\}$ exists. Because the right side of (B4) is continuous in $k_l$, we may define $N_{0^{**}}(x, z) = \lim_{k_l \to 0} N_{0^{***}}(x, z, k_l) = N_{0^{**}}(x, z, 0)$. Furthermore, substituting $N_0 = e^{-\mu_{\max} x}$ into the right side of (B4) when $k_l = 0$ yields $e^{-\mu_{\max} x}$, so because the left side exceeds $e^{-\mu_{\max} x}$, we must have $N_{0^{***}}(x, z, k_l) \geq N_{0^{**}}(x, z) > e^{-\mu_{\max} x} \quad (N_{0^{**}}$ decreases in $k_l)$. The left side of (B4) is less than or equal to the left side of (B1) as shown in the analysis after (B1), so $N_{0^{**}}(x, z, k_l) \leq N_{0^{*}}(x, z, k_l)$ and $N_{0^{***}}(x, z) \leq N_{0^{**}}(x, z)$.

We prove the remaining claims about the $xz > 0$ case for $N_0$ such that $e^{-\mu_{\max} x} < N_0 < N_{0^{***}}(x, z)$. Such $N_0$ satisfy $e^{-\mu_{\max} x} < N_0 < N_{0^{**}}(x, z, k_l)$ given the inequalities above. By the proof of Lemma B4, $p_0^s = p_1^i(e^{\mu_{\max}x}N_0) + k_l$ in equilibrium for such $N_0$. Assume for a contradiction that $(\alpha^{sell})^* L_1^* = 0$ for all developers. The largest possible sum of $L_1^*$ across all developers equals $\sum_{\theta > \theta_{\max}} L_0$. An equilibrium is possible only if the housing demand from potential residents is at least equal to the remaining land. This condition is

\[
\sum_{\theta < \theta_{\max}} L_0/S \leq \int_{\Theta} N_0 D(p_1^h(e^{\mu_{\max}x}N_0) + k_l - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta) d\theta,
\]

which fails for $N_0 < N_{0^{***}}(x, z)$ due to (B4), providing the necessary contradiction and proving that
the aggregate value of issued equity is positive.

From one of the developer constraints, \((L_0^{buy})^* - (H_0^{build})^* = L_1^* - L_0\), so the sum of the former across equity-issuing developers equals the sum of the latter across them. Assume for a contradiction that the latter sum is \(\leq 0\). For all developers not issuing equity, \(L_1^* \leq L_0\), with \(L_1^* = 0\) for developers without access to the equity market for whom \(\theta < \theta_i^{max}\). As a result, the total demand for space may equal \(S\) only if the precise opposite of (B4) holds. Because \(N_0 < N_0^{**}(x, z)\), we have a contradiction that proves that developers who issue equity in the aggregate buy land beyond construction needs.

We now prove the statement about shorting of equity-issuing developers. Pick any \(\theta' < \theta_i^{max}\) such that \(\int_{\theta' < \theta'} f_i(\theta) d\theta > 0\), where \(f_i\) is the distribution of \(\theta\) across equity investors (Assumption 4 guarantees the existence of \(\theta'\)). We will show that we can find \(k_s\) small enough so that \((\alpha^{buy})^* = -\pi\) for all \(\theta \leq \theta'\). If \(e^{\mu(\theta)x}N_0 \leq 1\), then \((\alpha^{buy})^* = -\pi\) if and only if \(k < (1 - k_s)ke^{\mu^{maxx}/\epsilon}N_0^{1/\epsilon} - k_s k_l\). As \(k_s \to 0\), the right side limits to something greater than the left as \(k_s\) goes to zero, so we can find \(k_s\) small enough so that \((\alpha^{buy})^* = -\pi\) for all \(\theta\) with \(e^{\mu(\theta)x}N_0 \leq 1\). Now consider \(\theta \leq \theta'\) with \(e^{\mu(\theta)x}N_0 > 1\). An equity investor with such \(\theta\) sets \((\alpha^{buy})^* = -\pi\) if and only if \(ke^{\mu(\theta)x}/\epsilon N_0^{1/\epsilon} - (1 - k_s)ke^{\mu^{maxx}/\epsilon}N_0^{1/\epsilon} - k_s k_l\). This equation holds if \(ke^{\mu(\theta)x}/\epsilon N_0^{1/\epsilon} < (1 - k_s)ke^{\mu^{maxx}/\epsilon}N_0^{1/\epsilon} - k_s k_l\). Because \(\theta' < \theta_i^{max}\), the right side limits to something greater than the left as \(k_s \to 0\), so we may choose \(k_s\) small enough so that the inequality holds. We may pick \(k_s\) small enough so that \((\alpha^{buy})^* = -\pi\) for all \(\theta \leq \theta'\), as desired.

From Lemma B2, the price of the claim on a developer for whom \((\alpha^{sell})^* > 0\) and \(L_1^* > 0\) equals 

\[
p_0^\pi = (ke^{\mu^{maxx}/\epsilon}N_0^{1/\epsilon} + k_l)L_1^*.
\]

This expression increases strictly in \(x\), as claimed. The price at \(t = 1\) equals 

\[
p_1^\pi = (p_1^*(e^{\mu^{truex}}N_0) + k_l)L_1^*\]

which is strictly less than \(p_0^\pi\) if and only if \(\mu^{true} < \mu_i^{max}\).

\[\square\]

### C Rental Extension

A share \(\chi \in [0, 1]\) of residents are of type \(a = 1\) and get flow utility only from renting; the remainder are of type \(a = 0\) and get flow utility only from owning.\(^2\) The type \(a\) is distributed independently from \(v\) and \(\theta\). All residents can act as landlords, but developers cannot (the developer problem remains the same as before). We denote \(R_t^{buy}\) the quantity of housing rented as a tenant and \(R_t^{sell}\) the quantity rented as a landlord. The rental price of housing equals \(p_t^r\). At \(t = 1\), an arriving potential resident chooses \((H_1^{buy})^*, (R_1^{buy})^*, (R_1^{sell})^*\) from

\[
\arg \max_{H_1^{buy}, R_1^{buy}, R_1^{sell}} \left( a t(H_1^{buy}) + (1 - a) t(H_1^{sell}) \right) v - p_1^h H_1^{buy} - p_1^r (R_1^{buy} - R_1^{sell})
\]

subject to

\[
\begin{align*}
0 &\leq H_1^{buy} \\
0 &\leq R_1^{buy} \\
0 &\leq R_1^{sell} \\
R_1^{sell} &\leq H_1^{buy},
\end{align*}
\]

\(^2\)We rule out \(\chi = 1\) because \(f_3^\pi\) does not satisfy Assumption 4 when \(\chi = 1\), meaning that the expressions \(p_0^b(N_0, x, z, f_2^\pi, f_4)\) and \(N_0^*(x, z, f_3^\pi)\) that appear in Proposition 8 are not well-defined. The existence of equilibrium does not depend on \(\chi \neq 1\), so by continuity the \(\chi = 1\) equilibrium equals the limiting equilibrium as \(\chi \to 1\).
where \( \iota(R) = 1 \) if \( R \geq 1 \) and 0 otherwise. The utility \( u(p^h_1, B_1, v, a, H^\text{buy}_{0}, R^\text{buy}_{0}, R^\text{sell}_{0}) \) at \( t = 1 \) of a potential resident of type \( a \) and \( v \) who arrived at \( t = 0 \) and chose \( H^\text{buy}_{0}, R^\text{buy}_{0}, \) and \( R^\text{sell}_{0} \) equals

\[
\arg \max_{H^\text{sell}_{1}} \left( \iota(R^\text{buy}_{0}) + (1 - a)\iota(R^\text{buy}_{0} - R^\text{sell}_{0}) \right) v + H^\text{sell}_{1} p^h_1 + B_1
\]

subject to \( 0 \leq H^\text{sell}_{1} \leq H^\text{buy}_{0} \).

At \( t = 0 \), arriving potential residents maximize the subjective expectation of their utility by choosing \( (H^\text{buy}_{0})^*, (R^\text{buy}_{0})^*, \) and \( (R^\text{sell}_{0})^* \) from

\[
\arg \max_{H^\text{buy}_{0}, R^\text{buy}_{0}, R^\text{sell}_{0}} \mathbb{E}[u(p^h_1, B_1, v, a, H^\text{buy}_{0}, R^\text{buy}_{0}, R^\text{sell}_{0})]
\]

subject to \( 0 \leq H^\text{buy}_{0} \)
\( 0 \leq R^\text{buy}_{0} \)
\( 0 \leq R^\text{sell}_{0} \)
\( R^\text{sell}_{0} \leq H^\text{buy}_{0} \)
\( B_0 = -p^h_0 H^\text{buy}_{0} - p^0_0 (R^\text{buy}_{0} - R^\text{sell}_{0}) \).

Equilibrium is the same as before with the addition of the condition that the sum of \( (R^\text{buy}_{t})^* \) across all residents equals the sum of \( (R^\text{sell}_{t})^* \) across them at each \( t \). The following lemma characterizes this equilibrium at \( t = 1 \):

**Lemma C1.** A unique equilibrium at \( t = 1 \) exists and coincides with that given by Lemma 1.

**Proof.** A potential resident arriving at \( t = 1 \) of type \( a = 1 \) gets utility \( v - p^r_1 \) from setting \( R^\text{buy}_{1} = 1 \) and utility 0 from setting \( R^\text{buy}_{1} = 0 \) (all other choices are dominated). The sum of \( (R^\text{buy}_{1})^* \) therefore equals \( \chi N_1 SD(p^h_1) \).

Increasing \( H^\text{buy}_{1} \) and \( R^\text{sell}_{1} \) the same amount increases utility of \( p^h_1 > p^h_1 \) and decreases utility if \( p^r_1 < p^h_1 \). The former cannot hold in equilibrium, as it leads to unlimited housing demand, which cannot be matched by the limited supply. The latter cannot hold in equilibrium if \( \chi > 0 \), as it leads to zero rental supply, which cannot be matched by rental demand, which is positive if \( \chi > 0 \). If \( \chi = 0 \), then \( p^r_1 = p^h_1 \) can hold in equilibrium if \( (R^\text{sell}_{0})^* = 0 \) for all potential residents. Therefore \( p^h_1 = p^r_1 \) or \( \chi = 0 \) and \( p^h_1 < p^r_1 \).

If \( \chi > 0 \), then a potential resident arriving at \( t = 1 \) of type \( a = 1 \) sets \( (H^\text{buy}_{1})^* = (R^\text{sell}_{1})^* \). Arriving potential residents of type \( a = 0 \) set \( (H^\text{buy}_{1})^* = 0 \) because \( p^r_1 = p^h_1 \) \( \geq k > 0 \) in the case that \( \chi > 0 \), or because clearing of the rental market in the case that \( \chi = 0 \) and \( p^r_1 < p^h_1 \) requires it (the equilibrium possibilities are then \( p^r_1 \in [0, p^h_1] \)). Setting \( R^\text{sell}_{1} = 0 \) in the case that \( \chi = 0 \) and \( p^r_1 < p^h_1 \), arriving potential residents of type \( a = 0 \) get utility \( v - p^h_1 \) if \( H^\text{buy}_{1} - R^\text{sell}_{1} = 0 \) and utility 0 if \( H^\text{buy}_{1} - R^\text{sell}_{1} = 0 \) (all other choices of \( H^\text{buy}_{1} - R^\text{sell}_{1} \) are dominated). The total of \( (H^\text{buy}_{1})^* - (R^\text{sell}_{1})^* \) across these potential residents equals \( (1 - \chi) N_1 SD(p^h_1) \).

The total of \( (H^\text{buy}_{1})^* - (R^\text{sell}_{1})^* + (R^\text{buy}_{1})^* \) across all residents equals \( N_1 SD(p^h_1) \). Because the rental market clears, the total of \( (H^\text{buy}_{1})^* \) equals \( N_1 SD(p^h_1) \), which coincides with housing demand in the model of Section 1. As shown in the proof of Lemma 1, the sum of \( (R^\text{sell}_{1})^* \) across departing residents is irrelevant for equilibrium prices at \( t = 1 \), so we are done.
We now prove Proposition 8.

**Proof of Proposition 8.** For clarity, we divide the proof into three parts.

**Part 1: Equilibrium house price at \( t = 0 \)**

Consider potential residents for whom \( a = 1 \). If \( R_{buy}^0 \in [0, 1) \), then utility is \( H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) - \theta_{buy}^0 + (R_{sell}^0 - R_{buy}^0)p_0^r \). If \( p_0^r < 0 \), then \( H_0^{buy} \) cannot be chosen to maximize utility, so \( p_0^r \geq 0 \) in equilibrium. As a result, utility weakly increases in \( R_{sell}^0 \), so it is maximized when \( R_{sell}^0 = H_0^{buy} \) and \( R_{buy}^0 = 0 \) at \( H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) + p_0^r - \theta_{buy}^0 \). If \( R_{buy}^0 \geq 1 \), then utility equals \( v + H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) - \theta_{buy}^0 + (R_{sell}^0 - R_{buy}^0)p_0^r \), which is maximized when \( R_{buy}^0 = 1 \) and \( R_{sell}^0 = H_0^{buy} \) at \( v - \theta_{sell}^0 + H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) + p_0^r - \theta_{buy}^0 \). Thus, unless \( p_0^r = 0 \) (which we consider below), the sum of \( (R_{sell}^0)^* \) across potential residents of type \( a = 1 \) equals \( \chi N_0 SD(p_0^r) \), and the sum of \( (R_{sell}^0)^* \) across them equals the sum of \( (H_0^{buy})^* \) across them.

Consider the problem for potential residents with \( a = 0 \). If \( H_0^{buy} - R_{sell}^0 \in [0, 1) \), then utility equals \( H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) - \theta_{buy}^0 \). Utility is maximized when \( R_{sell}^0 = H_0^{buy} \) at \( H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) + p_0^r - \theta_{buy}^0 \). If \( p_0^r(e^{\mu(x)}x)N_0) + p_0^r - \theta_{buy}^0 > 0 \) for any \( \theta \in supp f_r \), then utility cannot be maximized. As a result, \( p_1^h(e^{\mu(x)}x)N_0) + p_0^r - \theta_{buy}^0 \leq 0 \) for all \( \theta \in supp f_r \), and utility is maximized at \( 0 \). If \( H_0^{buy} - R_{sell}^0 \geq 1 \), then utility equals \( v + H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) - \theta_{buy}^0 \) at \( v - \theta_{sell}^0 + H_0^{buy}(p_1^h(e^{\mu(x)}x)N_0) + p_0^r - \theta_{buy}^0 \). Because \( p_1^h(e^{\mu(x)}x)N_0) + p_0^r - \theta_{buy}^0 \leq 0 \) for all \( \theta \in supp f_r \), utility is maximized when \( R_{sell}^0 = 0 \) and \( H_0^{buy} = 1 \) at \( v - \theta_{sell}^0 + p_1^h(e^{\mu(x)}x)N_0) \). Thus, unless \( p_0^r = 0 \), the sum of \( (R_{sell}^0)^* \) across these potential residents equals \( 0 \), and the sum of \( (H_0^{buy})^* \) across them exceeds the sum of \( (R_{sell}^0)^* \) across then by \( (1 - \chi)N_0 \int \Theta \! D(p_0^r - p_1^h(e^{\mu(x)}x)N_0)f_r(\theta)d\theta \).

Combining the two cases, we see that if \( p_0^r > 0 \), the sum of \( (H_0^{buy})^* \) across potential residents equals \( \chi N_0 SD(p_0^r) + (1 - \chi)N_0 \int \Theta \! D(p_0^r - p_1^h(e^{\mu(x)}x)N_0)f_r(\theta)d\theta \) due to the clearing of the rental market.

If \( p_1^h(e^{\mu max x}N_0) + p_0^r - \theta_{buy}^0 < 0 \), then \( (R_{sell}^0)^* = 0 \) for all potential residents. The rental market can clear only if \( \chi N_0 SD(p_0^r) = 0 \), which can hold only if \( \chi = 0 \). In this case, rental supply and demand equals 0 for all potential residents (or \( p_0^r = 0 \)), in which case the rental market becomes irrelevant and the equilibrium reduces to that analyzed in Section 3. Therefore, for the rest of the proof we assume that \( \chi > 0 \). In this case, \( (R_{sell}^0)^* > 0 \) for some potential residents, so \( p_1^h(e^{\mu max x}N_0) + p_0^r - \theta_{buy}^0 = 0 \).

As shown in the proof of Lemma 2, either \( p_0^h = k + p_1^h(e^{\mu max x}N_0) \) in which case developers for whom \( p_1^h(e^{\mu(x)}x)N_0) = p_1^h(e^{\mu max x}N_0) \) may choose any \( L_1 \geq 0 \), or \( p_0^h > k + p_1^h(e^{\mu max x}N_0) \) in which case \( L_1 = 0 \) for all developers. The former may hold in equilibrium if and only if the resulting housing demand falls short of \( S \):

\[
1 \geq \chi N_0 D(p_1^h(e^{\mu max x}N_0) - p_1^h(e^{\mu max x}N_0)) + (1 - \chi)N_0 \int \Theta \! D(p_0^r - p_1^h(e^{\mu(x)}x)N_0)f_r(\theta)d\theta.
\]

This inequality is the same as (A1) but with \( f_r \) replaced by \( f_r^* \), so (C1) holds if and only if \( N_0 \leq N_0^*(x, z, f_r^*) \). For such \( N_0 \), \( p_0^h = k + p_1^h(e^{\mu max x}N_0) = p_0^h(N_0, x, z, f_r^*, f_d) \). If \( N_0 > N_0^*(x, z, f_r^*) \), then \( p_0^h \) must equate total housing demand with \( S \), meaning that it is the unique value satisfying

\[
1 = \chi N_0 D(p_0^h - p_1^h(e^{\mu max x}N_0)) + (1 - \chi)N_0 \int \Theta \! D(p_0^r - p_1^h(e^{\mu(x)}x)N_0)f_r(\theta)d\theta.
\]
This equation coincides with (A2) but with \( f_\chi^r \) in place of \( f_r \), so the equilibrium house price at \( t = 0 \) equals \( p_0^h(N_0, x, z, f_\chi^r, f_d) \) for \( N_0 \geq N_r^*(x, z, f_\chi^r) \).

\[ \text{Part 2: Non-monotonicity of house price boom} \]

According to Proposition 5, the boom is strictly maximized at \( N_0 = 1 \) if Assumption 5 holds when applied to \( f_\chi^r \) in place of \( f_r \). The first condition in the assumption, \( \epsilon \mu_r^{\text{max}} x > \epsilon \mu_r^{\text{max}} x - 1 \), continues to hold because the maxima of \( f_r \) and \( f_\chi^r \) coincide. The second condition applied to \( f_\chi^r \) is

\[
1 > \chi \left( 1 + \epsilon \mu_r^{\text{max}} x - \epsilon \mu_r^{\text{max}} x / \epsilon \right) - (1 - \chi) \int_\Theta \left( 1 + \epsilon \mu_r^{\text{max}} x - \epsilon \mu_r^{\text{max}} x / \epsilon \right)^{-\epsilon} f_r(\theta) d\theta. \quad (C3)
\]

This inequality holds for \( \chi = 0 \) by assumption. The right side of (C3) increases continuously in \( \chi \) because \( \mu_r^{\text{max}} > \mu(\theta) \) for all \( \theta \in \Theta \), so (C3) holds for all \( \chi \) if and only if it holds for \( \chi = 1 \). When \( \chi = 1 \), (C3) reduces to \( \mu_r^{\text{max}} < \mu_r^{\text{max}} \). If \( \mu_r^{\text{max}} \geq \mu_r^{\text{max}} \), then by the intermediate value theorem there exists \( \chi^*(x, z) \in (0, 1] \) such that (C3) holds if \( \chi < \chi^*(x, z) \). When \( \mu_r^{\text{max}} = \mu_r^{\text{max}} \), (C3) holds as an equality, so \( \chi^*(x, z) = 1 \).

\[ \text{Part 3: House price boom and rental share} \]

As shown earlier in this proof,

\[
\frac{\sum(R_{\text{buy}}^h)^*}{\sum(H_{\text{buy}}^h)^*} = \frac{\chi D(p_0^h(N_0, x, z, \chi) - p_1^h(\epsilon \mu_r^{\text{max}} x N_0))}{\int_{\Theta} D(p_0^h(N_0, x, z, \chi) - p_1^h(\epsilon \mu(\theta) x N_0)) f_\chi^r(\theta) d\theta}.
\]

If \( xz = 0 \), then \( \mu(\theta) = \mu_r^{\text{max}} \) for all \( \theta \), so this fraction equals \( \chi \).

We now fix a value of \( N_0 \) and consider the case when \( xz > 0 \). The right side of (C1) weakly increases in \( \chi \), so \( N_r^*(x, z, f_\chi^r) \) weakly and continuously decreases in \( \chi \). As a result, if \( N_0 < N_r^*(x, z, \chi) \), then a marginal increase in \( \chi \) has no bearing on \( p_0^h(N_0, x, z, f_\chi^r, f_d) \), as this equilibrium price is independent of \( \chi \) for \( N_0 < N_r^*(x, z, f_\chi^r) \). If \( N_0 \geq N_r^*(x, z, f_\chi^r) \), then \( p_0^h(N_0, x, z, f_\chi^r, f_d) \) solves (C2). Because \( N_0 \geq N_r^*(x, z, f_\chi^r) > 1 \), the integral in (C2) evaluated at \( p_0^h = p_0^h(N_0, x, z, f_\chi^r, f_d) \) must be less than 1. It follows that \( D(p_0^h(N_0, x, z, \chi) - p_1^h(\epsilon \mu_r^{\text{max}} x N_0)) > \int_{\Theta} D(p_0^h(N_0, x, z, \chi) - p_1^h(\epsilon \mu(\theta) x N_0)) f_\chi^r(\theta) d\theta \), so an increase in \( \chi \) increases the right side of (C2) holding \( p_0^h = p_0^h(N_0, x, z, f_\chi^r, f_d) \) constant. Because the right side of (C2) weakly decreases in \( p_0^h \), it follows that \( p_0^h(N_0, x, z, f_\chi^r, f_d) \) strictly increases in \( \chi \), as desired.

\[ \square \]

\[ \text{D Supply Elasticity Extension} \]

Developers may rent out undeveloped land on spot markets each period to firms, such as banana stands, that use the city’s land as an input. We denote the land rent \( r_r^l \). Spot land demand of firms equals \( SD^{D_l}(r_r^l) \), where \( D_l \) satisfies the following:

**Assumption D1.** \( D_l : \mathbb{R}_+ \to \mathbb{R}_+ \) is continuously differentiable and decreases, \( -r(D_l)'(r)/D_l(r) \) weakly decreases, and \( \lim_{r \to 0} D_l(r) \geq 1 > \lim_{r \to -\infty} D_l(r). \)

The positivity of \( D_l \) guarantees that some vacant land exists in equilibrium, a property that makes analyzing the equilibrium easier. The condition on \( r(D_l)'/D_l \) means that land demand becomes weakly less elastic as its spot price rises so that it is weakly costlier to use each marginal unit of land. The first limit implies that land demand is at least equal to available space when land is free.
and leads to a positive spot price in equilibrium. The second limit implies that land demand falls below available space at a high enough price and leads to the existence of equilibrium.

Each developer chooses the quantity $L^t_{rent}$ of land to rent on the spot market. At $t = 1$, the liquidation value $\pi$ of a developer is the outcome of the constrained optimization problem

$$\pi(p^h_1, p^l_1, r^l_1, H_1, L_1, B_1) = \max_{H^sell_1, L^buy_1, H^build_1, L^rent_1} p^h_1 H^sell_1 - p^l_1 L^buy_1 - k H^build_1 + r^l_1 L^rent_1 + B_1$$

subject to

$$H^sell_1 \leq H^build_1$$

$$H^build_1 \leq L_1 + L^buy_1 - L^rent_1.$$ 

The actions $(H^sell_1)^*, (L^buy_1)^*, (H^build_1)^*$, and $(L^rent_1)^*$ chosen by the developer maximize this problem. At $t = 0$ each developer chooses $(H^sell_0)^*, (L^buy_0)^*, (H^build_0)^*$, and $(L^rent_0)^*$ from

$$\arg\max_{H^sell_0, L^buy_0, H^build_0, L^rent_0} E\pi(p^h_0, p^l_0, r^l_0, H_1, L_1, B_1)$$

subject to

$$H^sell_0 \leq H^build_0$$

$$H^build_0 \leq L_0 + L^buy_0 - L^rent_0$$

$$H^build_1 = H^build_0 - H^sell_0$$

$$L_1 = L_0 + L^buy_0 - H^build_0$$

$$B_1 = p^h_0 H^sell_0 - p^l_0 L^buy_0 - 2k H^build_0 + r^l_1 L^rent_1.$$ 

The potential resident problems are the same as in Appendix C. Equilibrium is the same as before with the addition of the condition that the sum of $(L^rent_t)^*$ across developers equals $SD^t(r^l_t)$ at each $t$. The following lemma characterizes this equilibrium at $t = 1$:

**Lemma D1.** Given $N_1$, a unique equilibrium at $t = 1$ exists, and in it $p^h_1 - k = p^l_1 = r^l_1 > 0$.

**Proof.** If $r^l_1 \neq p^l_1$, then developers cannot maximize $\pi$ because holding $L^build_1 - L^rent_1$ constant and increasing $L^buy_1$ and $L^rent_1$ always increases $\pi$ if $r^l_1 > p^l_1$ and decreases $\pi$ if $r^l_1 < p^l_1$. So $p^l_1 = r^l_1$ in any equilibrium. For the same reasons given in the proof of Lemma 1, $p^h_1 = p^l_1 + k$ in any equilibrium.

From the proof of Lemma C1, housing demand from arriving potential residents at $t = 1$ equals $SN_1 D(p^h_1)$. Land demand from firms equals $SD^t(r^l_1)$. If $r^l_1 \leq 0$, then demand for space is either not defined or exceeds $S$. Therefore, in any equilibrium $r^l_1 > 0$. It follows that $(L^rent_1)^* + (H^build_1)^* = L_1 + (L^buy_1)^*$ for all developers. Similarly, $(H^sell_1)^* = H_1 + (H^build_1)^*$ for all developers. Therefore the sum of $(H^sell_1)^* + (L^rent_1)^*$ across developers equals the sum of $H_1 + L_1$ across them. All space other than $H_1 + L_1$ is owned by departing potential residents at the beginning of $t = 1$. The clearing of the land spot market and the housing market therefore imply that in equilibrium, $1 = D^t(r^l_1) + N_1 D(p^h_1)$. Substituting $r^l_1 = p^h_1 - k$ yields

$$1 = D^t(p^h_1 - k) + N_1 D(p^h_1). \quad (D1)$$

Because $r^l_1 > 0$, $p^h_1 > k$, so both $D^t$ and $D$ strictly decrease for possible $p^h_1$. The right side exceeds 1 as $p^h_1 \to k$. As $p^h_1 \to \infty$, the right side limits to something less than 1 by Assumption D1. It follows that a unique value of $p^h_1$ satisfies this equation.

We denote equilibrium prices $p^h_1(N_1)$, $p^l_1(N_1)$, and $r^l_1(N_1)$. The first two should not be confused with the functions defined after Lemma 1 that use the same notation.

The next lemma establishes the existence of a unique equilibrium at $t = 0$. 

Lemma D2. Given \( N_0, x, z, \) and \( \chi, \) a unique equilibrium at \( t = 0 \) exists.

Proof. For the same reasons given in the proof of Lemma D1, \( r_0^h > 0 \) in any equilibrium. In the equilibrium at \( t = 1, \pi = p^h_1 H_1 + p^l_1 L_1 + B_1. \) By making substitutions using the constraints of the \( t = 0 \) developer problem, we see that the objective at \( t = 0 \) is to choose \( H_1, L_1 \geq 0 \) to maximize

\[
(p^h_1(e^{\mu (\theta)x}N_0) - p^h_0)H_1 + (p^l_1(e^{\mu (\theta)x}N_0) - p^l_0 + k + r_0^l)L_1 + p^h_0 L_0 \quad \text{and} \quad (L_0^{\text{rent}})^* = L_1 \text{ for each developer. Because } k + r_0^l > 0, \text{ it follows that } H_1 = 0 \text{ for all developers. If } p^h_1(e^{\mu (\theta)x}N_0) - p^l_0 + k + r_0^l < 0 \text{ for all developers, then } L_1 = 0 \text{ for all of them, but then } (L_0^{\text{rent}})^* = 0, \text{ leading to a failure of market-clearing in the land spot market because } D^l(r_0^l) > 0. \text{ If } p^h_1(e^{\mu (\theta)x}N_0) - p^l_0 + k + r_0^l > 0 \text{ for any developer, then the objective function cannot be maximized. It follows that } p^h_0 = p^l_1(e^{\mu (\theta)x}N_0) + k + r_0^l. \text{ Market-clearing in all markets implies that spot land demand plus total housing demand from arriving potential residents equals } S. \text{ Using the equations for housing demand from the proof of Proposition 8, we form the equilibrium condition}

\[
1 = D^l(p^h_0 - p^h_1(e^{\mu (\theta)x}N_0) - k) + \chi N_0 D(p^h_0 - p^h_1(e^{\mu (\theta)x}N_0)) + (1 - \chi) N_0 \int_\Theta D(p^h_0 - p^h_1(e^{\mu (\theta)x}N_0)) f_r(\theta) d\theta. \tag{D2}
\]

The right side strictly decreases in \( p^h_0 \) wherever it is defined. It is defined for \( p^h_0 > k + p^h_1(e^{\mu (\theta)x}N_0). \) As \( p^h_0 \) approaches this value, \( D^l \) is at least 1, whereas the remainder of the right side is positive. It follows that the entire right side exceeds 1 in the limit. As \( p^h_0 \to \infty, \) the terms involving \( D \) go to 0, and the term involving \( D^l \) limits to something less than 1 according to Assumption D1. It follows that a unique solution exists to this equation.

We denote this unique equilibrium price \( p^h_0(N_0, x, z, \chi), \) which should not be confused with the equilibrium price given by Proposition 8.

We turn now to defining the elasticity of housing supply. The proof of Lemma C1 showed that \( r^h_1 = p^h_1(N_1) \) is the unique equilibrium rent when \( \chi > 0 \) and is an equilibrium rent when \( \chi = 0. \) We define \( r^h_0(N_1) = p^h_1(N_1). \) Because the housing stock at \( t = 1 \) equals \( S - S D^l(p^h_1 - k), \) the elasticity of housing supply at \( t = 1 \) is

\[
\epsilon^h_1(N_1) = \frac{r^h_1(N_1)(D^l)'(p^h_1(N_1) - k)}{1 - D^l(p^h_1(N_1) - k)}.
\]

Similarly, the proof of Proposition 8 showed that \( r^h_0 = p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu (\theta)x}N_0) \) is the unique equilibrium rent when \( \chi > 0 \) and is an equilibrium rent when \( \chi = 0. \) We define \( r^h_0(N_0, x, z, \chi) = p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu (\theta)x}N_0). \) Because the housing stock equals \( S - S D^l(p^h_0 - p^h_1(e^{\mu (\theta)x}N_0) - k) \) at \( t = 0, \) the elasticity of housing supply at \( t = 0 \) equals

\[
\epsilon^h_0(N_0, x, z, \chi) = \frac{r^h_0(N_0, x, z, \chi)(D^l)'(p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu (\theta)x}N_0) - k)}{1 - D^l(p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu (\theta)x}N_0) - k)}.
\]

The next lemma characterizes these elasticities.

Lemma D3. There exists a continuous, decreasing function \( \epsilon^h : \mathbb{R}_+ \to \mathbb{R}_+ \) such that \( \epsilon^h_0(N_0, x, 0, \chi) = \epsilon^h(N_0) \) and \( \epsilon^h_1(N_1) = \epsilon^h(N_1). \)

Proof of Lemma D3. We define the function \( \epsilon^h(\cdot) \) by

\[
\epsilon^h(N) \equiv \frac{p^h_1(N)(D^l)'(p^h_1(N) - k)}{1 - D^l(p^h_1(N) - k)}. \tag{D3}
\]
Given (D1), the denominator equals $ND(p^h_1(N)) > 0$. As shown by Lemma D1, $p^h_1(N) > k$, so the numerator is negative and well-defined. It follows that $e^s(N) > 0$ for all $N > 0$. Because $p^h_1(N) > k$, the implicit function theorem applied to (D1) implies that $p^h_1(\cdot)$ is continuous; Assumption D1 then implies that $e^s(\cdot)$ is continuous. To show that $e^s$ decreases, we rewrite (D3) as

$$e^s(N) = \frac{-(p^h_1(N) - k)(D'(p^h_1(N) - k) p^h_1(D(p^h_1(N) - k)) \frac{D(p^h_1(N) - k)}{p^h_1(N) - k}}{1 - D(p^h_1(N) - k)}$$

(D4)

It is clear from (D1) that $p^h_1(N)$ strictly increases in $N$ because $D'$ and $D$ both strictly decrease over the domains relevant in that equation. It follows that each fraction on the right of (D4) strictly decreases in $N$, with the result about first fraction following from Assumption D1. Because $r^h_1(N_1) = p^h_1(N_1)$, $\epsilon^s = e^s(N_1)$. When $z = 0$, it is clear from (D1) that $p^h_1 = p^h_1(N_0) + p^h_1(e^{\mu x} N_0)$ solves (D2). Therefore $r^h_1(N_0, x, 0, \chi) = p^h_1(N_0)$ and $p^h_0(N_0, x, 0, \chi) - p^h_1(e^{\mu x} N_0) = p^h_1(N_0)$ when $z = 0$. it follows that $e^s_0(N_0, x, 0, \chi) = e^s(N_0)$.

Next, we prove Proposition 9. Differentiating (D1) and simplifying yields

$$\frac{\partial p^h_1(e^{\mu \theta} x N_0)}{\partial x} = \frac{\mu(\theta)p^h_1(e^{\mu \theta} x N_0)}{e^s(e^{\mu \theta} x N_0) + \epsilon}.$$

Using this equation, we differentiate (D2) with respect to $x$ and simplify to obtain

$$\frac{\partial \log p^h_0(N_0, x, z, \chi)}{\partial x} = \frac{\mu_{\mu_{\mu_{\max}}} p^h_0(N_0, x, z, \chi) \epsilon^s(e^{\mu_{\max} x} N_0) + \epsilon}{\epsilon(\epsilon(e^{\mu_{\max} x} N_0) + \epsilon)} + \int \frac{\epsilon_0 p^h_0(N_0, x, z, \chi) \epsilon(\epsilon(e^{\mu_{\max} x} N_0) + \epsilon)}{\epsilon(\epsilon(e^{\mu_{\max} x} N_0) + \epsilon)} f_0(\theta) d\theta,$$

where $c_i = \gamma_i/(\gamma_1 + \gamma_2 + \gamma_3)$ for $i \in \{1, 2, 3\}$ and the $\gamma_i$ are defined as follows:

$$\gamma_1 = (D')_1(p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu_{\max} x} N_0))$$

$$\gamma_2 = \epsilon_0 D'(p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu_{\max} x} N_0))$$

$$\gamma_3 = (1 - \epsilon_0) \epsilon_0 \int \epsilon(D(p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu \theta} N_0)) f_0(\theta) d\theta.$$

We now prove the equations in the proposition. When $x = 0$, $r^h_0(N_0, x, z, \chi) = p^h_0(N_0, x, z, \chi) - p^h_1(e^{\mu_{\max} x} N_0)$ for all $\theta$, so $\gamma_1 = e^s(N_0)/(\epsilon^s(N_0) + \epsilon)$. $\gamma_2 = \epsilon^s(N_0)/(\epsilon^s(N_0) + \epsilon)$, and $\gamma_3 = (1 - \epsilon^s(N_0))/(\epsilon^s(N_0) + \epsilon)$. We also have $p^h_0(N_0, 0, z, \chi) = p^h_0(N_0)/2$. It follows that

$$\frac{\partial \log p^h_0(N_0, 0, z, \chi)}{\partial x} = \frac{1}{2} \frac{e^s(N_0) \mu_{\max} + \epsilon \mu_{\max} + (1 - \epsilon)(\mu_{\max} N_0)}{\epsilon^s(N_0) + \epsilon},$$

which coincides with the formula in the text. When $z = 0$, $\mu(\theta) = \overline{\mu}$ for all $\theta$ and $p^h_0(N_0, x, z, \chi) = p^h_1(N_0) + p^h_1(e^{\mu \theta} N_0)$ as shown in the previous proof. It follows that

$$\frac{\partial \log p^h_0(N_0, 0, 0, \chi)}{\partial x} = \frac{p^h_1(e^{\overline{\mu} \theta} N_0)}{p^h_1(N_0) + p^h_1(e^{\overline{\mu} \theta} N_0)} \frac{\epsilon^s(e^{\overline{\mu} \theta} N_0) + \epsilon}{\epsilon^s(e^{\overline{\mu} \theta} N_0) + \epsilon}.$$
This expression coincides with the formula in the text because

\[
\frac{p_1^h(e^{\pi x}N_0)}{p_1^h(N_0)} = \exp \left( \int_0^x \frac{\partial \log p_1^h(e^{\pi x'}N_0)}{\partial x'} dx' \right) = \exp \left( \int_0^x \frac{\pi dx'}{e^{\pi x'} N_0} + \epsilon \right).
\]

Figure D1 plots the \( t = 0 \) pass-through \( 1/(e^\epsilon N_0) + \epsilon \) and \( t = 1 \) pass-through \( 1/(e^\epsilon (e^{\pi x} N_0)) + \epsilon \) as well as the approximation for \( \partial \log p_1^h(N_0, x, z, \chi) / \partial x \) with and without disagreement. Disagreement amplifies the price impact of \( x \) most when the short-run elasticity is high and the long-run elasticity is low.

\[\text{Lemma E1.} \quad Q_r(N_0, x, z) < Q_r(N_0, 0, z) \text{ if } e^{-\mu_d^{\max} x} < N_0 < N_0^* (x, z) \text{ and } z = 0. \quad Q_r(N_0, x, z) = Q_r(N_0, 0, z) \text{ otherwise.} \]

\[\text{Proof.} \quad \text{By Proposition 2, } Q_r(N_0, x, z) = 1 \text{ when } N_0 \geq N_0^*(x, z). \text{ By (A1) in the proof of Lemma 2, } Q_r(N_0, x, z) = \text{SN} \int_\Theta D(p_1^h(e^{\mu_d^{\max} x} N_0) + k - p_1^h(e^{\mu(\theta) x} N_0))) f_r(\theta) d\theta \text{ when } N_0 < N_0^*(x, z). \]

When \( z = 0 \), \( p_1^h(e^{\mu_d^{\max} x} N_0) = p_1^h(e^{\mu(\theta) x} N_0) \) for all \( \theta \in \Theta \), so \( Q_r(N_0, x, 0) = \text{SN} N_0 \text{ for } N_0 < N_0^*(x, 0) \). Because \( N_0^*(x, 0) = 1 \) as shown by Proposition 1, \( Q_r(N_0, x, 0) = Q_r(N_0, 0, 0) \).

When \( z > 0 \) and \( N_0 \leq e^{-\mu_d^{\max} x} \), \( S N_0 \int_\Theta (e^{\mu(\theta) x} N_0) \geq p_1^h(e^{\mu_d^{\max} x} N_0) \) for all \( \theta \in \Theta \), so by Assumption 1 \( Q_r(N_0, x, z) = \text{SN} N_0 \). Thus \( Q_r(N_0, x, z) = Q_r(N_0, 0, z) \) in this case as well.

When \( z > 0 \) and \( N_0 \geq N_0^*(x, z) \), \( Q_r(N_0, x, z) = \text{SN} \int_\Theta D(p_1^h(e^{\mu_d^{\max} x} N_0) + k - p_1^h(e^{\mu(\theta) x} N_0))) f_r(\theta) d\theta \).

We divide the final case in which \( e^{-\mu_d^{\max} x} < N_0 < N_0^*(x, z) \) and \( z > 0 \) into two subcases. If \( 1 \leq N_0 < N_0^*(x, z) \), then \( 1 = N_0^*(0, z) \leq N_0 < N_0^*(x, z) \). It follows that \( Q_r(N_0, x, z) = 1 = Q_r(N_0, 0, z) \), as claimed. If \( e^{-\mu_d^{\max} x} < N_0 < 1 \), then \( Q_r(N_0, 0, z) - Q_r(N_0, x, z) = \text{SN} \int_{\theta < \theta_d^{\max}} (1 - D(p_1^h(e^{\mu_d^{\max} x} N_0) + k - p_1^h(e^{\mu(\theta) x} N_0))) f_r(\theta) d\theta \).

By Assumption 4, \( \int_{\theta < \theta_d^{\max}} f_r(\theta) d\theta > 0 \), so \( Q_r(N_0, 0, z) > Q_r(N_0, x, z) \), as claimed.

Lemma E1 shows that the shock \( x \) only affects the equilibrium quantity of housing in intermediate cities with disagreement, in which case the shock lowers the housing stock. Because the shock does not change the current demand \( N_0 \), it does not alter housing supply in most cases. It only does so when optimistic developers set prices so high that the number of potential residents choosing to buy falls. This scenario occurs in intermediate cities with disagreement.
FIGURE D1
Comparative Statics with Respect to Initial Demand

a) Supply Elasticity

\[ (\epsilon^t + \epsilon)^{-1} \]

\[ N_0 \]

\[ t = 0 \quad t = 1 \]

b) Price Increase

\[ \text{Pass-Through of } x \text{ to } \log \hat{p} \]

\[ N_0 \]

Notes: \( N_0 \) equals the number of potential residents at \( t = 0 \) relative to the city size. Supply elasticities are given by \( \epsilon_0^t = \epsilon^t(N_0) \) and \( \epsilon_1^t = \epsilon^t(e^{\mu x N_0}) \), where \( \epsilon^t(\cdot) \) is defined by Lemma D3. The pass-through of \( x \) to \( \log \hat{p} \) equals the expression for \( \partial \log \hat{p}(N_0, x, z, \chi) / \partial x \) given by Proposition 9. The parameters used to generate this figure are \( k = 1, x = 1, z = 1, \epsilon = 1, \chi = 0, \mu = 1, f_r = f_d = (0.9)1_{-1/9} + (0.1)1_1, \) and \( D^t(r) = 0.01k/r, \) with \( z = 0 \) used in the “without disagreement” graph.
FIGURE E1
Land Supply Slides from Pulte’s 2004 Investor Conference

TABLE E1
Annualized Real House Price Growth, 2000-2006

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<th>(1)</th>
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<tr>
<td><strong>Anomalous City</strong></td>
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<tr>
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<td>(0.0082)</td>
<td>(0.0080)</td>
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*Elasticity-adjusted demand controls*

<p>| | | |</p>
<table>
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<tr>
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<tr>
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<td>9-12th grade, no diploma</td>
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<td>$R^2$</td>
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</table>

Notes: This table presents estimates of $\alpha$ and $\beta$ from the equation

$$\Delta \log p^h_j = \alpha 1_{j \in \{\text{Anomalous Cities}\}} + \frac{\beta d_j + \eta_j}{\epsilon_j^s + \epsilon},$$

where $j$ indexes metropolitan areas and $\Delta \log p^h_j$ equals the 2000-2006 annualized log change in the second quarter FHFA house price index deflated by the CPI-U. The Anomalous Cities are metro areas in our sample in Arizona, inland California, Florida, and Nevada. In specification (1), $d_j$ includes just a constant; in specification (2), $d_j$ further includes the listed demographics from the 2000 US Census, which are measured in shares of the population except for log population and log income. We set $\epsilon = 0.6$ and take the housing supply elasticity $\epsilon_j^s$ from Saiz (2010). We estimate $\alpha$ and $\beta$ by multiplying each side of the above equation by $\epsilon_j^s + \epsilon$ and then performing OLS with $\eta_j$ as the error term. Standard errors are in parentheses. Significance levels 10%, 5%, and 1% are denoted respectively by *, **, and ***.