Internet Appendix for "Arrested Development: Theory and Evidence of Supply-Side Speculation in the Housing Market"

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This Internet Appendix extends the main model in the ways discussed in Section IV of the paper, proves the propositions in Section IV, and provides additional theoretical and empirical analysis referenced in Section V.C of the paper.

I. Equity Extension

Developers who can access the equity market choose a share $\alpha^{sell} \in [0,1]$ of the claim to their total $t=1$ liquidation value to sell at $t=0$. The price of this claim equals p_0^{π}, which may vary across developers. Each of these developers may also pay itself a dividend δ at $t=0$ using its available cash flow. Finally, land that remains undeveloped at the end of $t=0$ pays a dividend $k_l > 0$ at $t=1$; we focus on the limiting equilibria as $k_l \to 0$.[1] The optimal behavior for such a developer is to choose δ^*, $(\alpha^{sell})^*$, $(H_0^{sell})^*$, $(L_0^{buy})^*$, and $(H_0^{build})^*$ from

$$\begin{aligned}
\underset{\delta,\alpha^{sell},H_0^{sell},L_0^{buy},H_0^{build}}{\arg\max} \quad & \delta + (1-\alpha^{sell})\mathrm{E}\pi(p_1^h, p_1^l, H_1, L_1, B_1) \\
\text{subject to} \quad & \alpha^{sell} \in [0,1] \\
& H_0^{sell} \leq H_0^{build} \\
& H_0^{build} \leq L_0 + L_0^{buy} \\
& H_1 = H_0^{build} - H_0^{sell} \\
& L_1 = L_0 + L_0^{buy} - H_0^{build} \\
& B_1 = p_0^h H_0^{sell} - p_0^l L_0^{buy} - 2kH_0^{build} + \alpha^{sell} p_0^{\pi} - \delta \\
& 0 \leq B_1 \\
& 0 \leq \delta.
\end{aligned}$$

Developers who cannot access the equity market face the same problem with the additional constraint $\alpha^{sell} = 0$. For all developers, the $t=1$ problem remains the same as before.

A unit measure of equity investors chooses a share α^{buy} of the claim to each developer's $t=1$ liquidation value to buy at $t=0$. The chosen α^{buy} may differ for each investor-developer pair. Each investor faces a proportional cost $k_s \in (0,1)$ for each dollar invested in

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[1] This dividend leads to a positive land price at $t=0$ that guarantees the existence of equilibrium when $\mathrm{E}p_1^l = 0$ for all equity investors but $\mathrm{E}p_1^l > 0$ for some developers. The proof of Proposition 7 further discusses this issue.

a negative position, and the most negative position that can be taken is $-\overline{\alpha}$, where $\overline{\alpha} > 0$. For a given developer, an equity investor chooses $(\alpha^{buy})^*$ from

$$\begin{aligned} \underset{\alpha^{buy}}{\arg\max} \quad & \alpha^{buy}\mathrm{E}\pi(p_1^h, p_1^l, H_1, L_1, B_1) - \max(\alpha^{buy}, (1-k_s)\alpha^{buy})p_0^\pi \\ \text{subject to} \quad & -\overline{\alpha} \leq \alpha^{buy} \\ & H_1 = (H_0^{build})^* - (H_0^{sell})^* \\ & L_1 = L_0 + (L_0^{buy})^* - (H_0^{build})^* \\ & B_1 = p_0^h(H_0^{sell})^* - p_0^l(L_0^{buy})^* - 2k(H_0^{build})^* + (\alpha^{sell})^* p_0^\pi - \delta^*, \end{aligned}$$

where E denotes the equity investor's expectation and δ^*, $(\alpha^{sell})^*$, $(H_0^{sell})^*$, $(L_0^{buy})^*$, and $(H_0^{build})^*$ denote the actions chosen by the developer.

The potential resident problems remain the same. Prices p_0^h, p_0^l, and p_0^π constitute an equilibrium when, in addition to the clearing of land and housing markets described in Section I, the following holds: for each developer, $(\alpha^{sell})^*$ equals the sum across equity investors of $(\alpha^{buy})^*$.

We now characterize equilibrium. The first lemma simplifies the objective of each developer.

LEMMA IA1: *In equilibrium, each developer chooses* α^{sell} *and* $L_1 \geq 0$ *such that* $p_0^l(L_0 - L_1) + \alpha^{sell}p_0^\pi \geq 0$ *to maximize* $p_0^l(L_0 - L_1) + \alpha^{sell}p_0^\pi + (1-\alpha^{sell})\mathrm{E}(p_1^l + k_l)L_1$.

Proof: In all of the $t = 1$ equilibria characterized in the proof of Lemma 1, $\pi = p_1^h H_1 + (p_1^l + k_l)L_1 + B_1$ (p_1^l is the ex-dividend price). At $t = 0$, the developer maximizes $\delta + (1 - \alpha^{sell})\mathrm{E}(p_1^h H_1 + (p_1^l + k_l)L_1 + B_1)$. From substituting the H_1 and L_1 constraints into the B_1 constraint, we have $B_1 = -p_0^h H_1 + p_0^l(L_0 - L_1) + (p_0^h - p_0^l - 2k)H_0^{build} + \alpha^{sell}p_0^\pi - \delta$. In equilibrium $p_0^h = p_0^l + 2k$, for otherwise each developer would want to build a positively or negatively infinite amount of housing. Therefore, $B_1 = -p_0^h H_1 + p_0^l(L_0 - L_1) + \alpha^{sell}p_0^\pi - \delta$. The developer maximizes $\delta + (1-\alpha^{sell})\mathrm{E}((p_1^h - p_0^h)H_1 + (p_1^l + k_l - p_0^l)L_1 + p_0^l L_0 + \alpha^{sell}p_0^\pi - \delta)$ by choosing $H_1, L_1 \geq 0$, $\alpha^{sell} \in [0, 1]$, and δ such that $B_1 \geq 0$. Because $p_1^h - p_0^h = p_1^l - p_0^l - k < p_1^l + k_l - p_0^l$, in equilibrium all developers set $H_1 = 0$ (if $H_1 > 0$ is optimal, then the developer wants an infinite L_1). The objective weakly increases in δ for $\alpha^{sell} \in [0, 1]$, so it is maximized at $\delta = -p_0^h H_1 + p_0^l(L_0 - L_1) + \alpha^{sell}p_0^\pi$, the largest possible value given the $B_1 \geq 0$ constraint. The $\delta \geq 0$ constraint produces $p_0^l(L_0 - L_1) + \alpha^{sell}p_0^\pi \geq 0$. The objective simplifies to $p_0^l(L_0 - L_1) + \alpha^{sell}p_0^\pi + (1-\alpha^{sell})\mathrm{E}(p_1^l + k_l)L_1$, as claimed. $\square$

The developer objective consists of three terms: profits from current land sales, revenues from equity offerings, and profits expected at $t = 1$ from end-of-period land holdings. The next lemma delivers the equilibrium price of equity.

LEMMA IA2: *In equilibrium,* $p_0^\pi = (p_1^l(e^{\mu_i^{max}x}N_0) + k_l)L_1$ *for any developer for whom* $(\alpha^{sell})^* > 0$.

Proof: As shown in the proof of Lemma IA1, each developer sets $H_1 = 0$ and sets $B_1 = 0$ when $\alpha^{sell} > 0$. The liquidation value of the developer becomes $\pi = (p_1^l + k_l)L_1$. If $p_0^\pi < (p_1^l(e^{\mu_i^{max}x}N_0) + k_l)L_1$, then the equity investors for whom $\theta = \theta_i^{max}$ want to set α^{buy} arbitrarily large. The equity market cannot clear in this case because the maximal aggregate short position across equity investors is bounded at $-\overline{\alpha}$. Therefore, $p_0^\pi \geq (p_1^l(e^{\mu_i^{max}x}N_0) + k_l)L_1$. If this inequality is strict, then $(\alpha^{buy})^* \leq 0$ for all equity investors, preventing clearing in the equity market. The only equilibrium outcome is the one given in the lemma. □

The price of any traded claim equals the most optimistic equity investor valuation of the land held by that developer at the end of $t = 0$. In this sense, traded developers act like land hedge funds by raising equity against speculative land investments. To make this point clear, the following lemma relates the equilibrium prices of developer equity and the land they hold:

LEMMA IA3: *In equilibrium,* $p_0^\pi = p_0^l L_1$ *for any developer for whom* $(\alpha^{sell})^* > 0$.

Proof: We prove this claim by delineating all possible choices by developers in equilibrium. By substituting Lemma IA2 into Lemma IA1, we rewrite the developer problem as choosing

$$\begin{aligned} L_1^*, (\alpha^{sell})^* \in & \arg\max_{L_1, \alpha^{sell}} \quad p_0^l L_0 + \left(\alpha^{sell} p_1^l(e^{\mu_i^{max}x}N_0) + (1-\alpha^{sell})p_1^l(e^{\mu(\theta)x}N_0) + k_l - p_0^l\right) L_1 \\ & \text{subject to} \quad p_0^l L_1 \leq p_0^l L_0 + \alpha^{sell}(p_1^l(e^{\mu_i^{max}x}N_0) + k_l)L_1 \\ & \qquad\qquad 0 \leq L_1 \\ & \qquad\qquad \alpha^{sell} \in [0,1] \text{ (with access to equity market)} \\ & \qquad\qquad \alpha^{sell} = 0 \quad \text{(without access to equity market).} \end{aligned}$$

A developer that cannot access the equity market sets $(\alpha^{sell})^* = 0$ and chooses

$$\begin{aligned} L_1^* &= L_0 & &\text{if } p_0^l < p_1^l(e^{\mu(\theta)x}N_0) + k_l \\ L_1^* &\in [0, L_0] & &\text{if } p_0^l = p_1^l(e^{\mu(\theta)x}N_0) + k_l \\ L_1^* &= 0 & &\text{if } p_0^l > p_1^l(e^{\mu(\theta)x}N_0) + k_l \end{aligned}$$

if $p_0^l > 0$. If $p_0^l \leq 0$, then L_1^* does not exist because the developer always increases its objective function without violating the constraints by increasing L_1 beyond L_0. Similarly, if $p_0^l < p_1^l(e^{\mu_i^{max}x}N_0) + k_l$, then L_1^* does not exist for developers with access to the equity market. With $\alpha^{sell} = 1$, increasing L_1 always increases the objective function while obeying the constraints. If $p_0^l = p_1^l(e^{\mu_i^{max}x}N_0) + k_l$, then the optimal choices for developers with access to the equity market are

$$\left.\begin{aligned} L_1^* &= \frac{L_0}{1-(\alpha^{sell})^*} \qquad \text{and } (\alpha^{sell})^* \in [0,1) \end{aligned}\right\} \text{if } p_1^l(e^{\mu_i^{max}x}N_0) < p_1^l(e^{\mu(\theta)x}N_0)$$

$$\left.\begin{aligned} & L_1^* \geq 0 \qquad\qquad\qquad \text{and } (\alpha^{sell})^* = 1 \\ & \quad \text{or} \\ & L_1^* \in \left[0, \frac{L_0}{1-(\alpha^{sell})^*}\right] \quad \text{and } (\alpha^{sell})^* \in [0,1) \end{aligned}\right\} \text{if } p_1^l(e^{\mu_i^{max}x}N_0) = p_1^l(e^{\mu(\theta)x}N_0)$$

$$\left.\begin{array}{ll} L_1^* = 0 & \text{and } (\alpha^{sell})^* \in [0,1) \\ \quad\text{or} & \\ L_1^* \geq 0 & \text{and } (\alpha^{sell})^* = 1 \end{array}\right\} \text{if } p_1^l(e^{\mu_i^{max}x}N_0) > p_1^l(e^{\mu(\theta)x}N_0).$$

The first case follows because if $\alpha^{sell} < 1$, the objective strictly increases in L_1 and so is maximized at $L_1^* = L_0/(1-\alpha^{sell})$ with a value of $(p_1^l(e^{\mu(\theta)x}N_0) + k_l)L_0$. This value exceeds $p_0^l L_0$, the objective function value obtained when $\alpha^{sell} = 1$. In the second case of the optimal developer choices, the objective is independent of L_1 and α^{sell}, so the developer may choose any feasible combination. In the third case, the objective decreases in L_1 if $\alpha^{sell} < 1$, leading to $L_1^* = 0$; if $\alpha^{sell} = 1$, then the objective is independent of L_1, permitting the developer to choose any feasible value for L_1^*. Finally, if $p_0^l > p_1^l(e^{\mu_i^{max}x}N_0) + k_l$, then

$$\begin{array}{lll} L_1^* = L_0 & \text{and } (\alpha^{sell})^* = 0 & \Big\} \text{ if } p_0^l < p_1^l(e^{\mu(\theta)x}N_0) + k_l \\[1ex] \left.\begin{array}{l} L_1^* \in [0, L_0] \\ \quad\text{or} \\ L_1^* = 0 \end{array}\right. & \left.\begin{array}{l} \text{and } (\alpha^{sell})^* = 0 \\ \\ \text{and } (\alpha^{sell})^* \in [0,1] \end{array}\right\} & \text{if } p_0^l = p_1^l(e^{\mu(\theta)x}N_0) + k_l \\[1ex] L_1^* = 0 & \text{and } (\alpha^{sell})^* \in [0,1] \Big\} & \text{if } p_0^l > p_1^l(e^{\mu(\theta)x}N_0) + k_l \end{array}$$

are the optimal choices for developers with access to the equity market. In the first case, the value of the objective function at the given choices equals $(p_1^l(e^{\mu(\theta)x}N_0) + k_l)L_0$. For $\alpha^{sell} \geq (p_1^l(e^{\mu(\theta)x}N_0) + k_l - p_0^l)/(p_1^l(e^{\mu(\theta)x}N_0) - p_1^l(e^{\mu_i^{max}x}N_0))$, the coefficient in the objective function on L_1 is nonpositive, meaning that it is maximized at $L_1^* = 0$ with a value of $p_0^l L_0$, which is less than the maximized value when $L_1^* = L_0$ and $(\alpha^{sell})^* = 0$. For $\alpha^{sell} \in (0, (p_1^l(e^{\mu(\theta)x}N_0) + k_l - p_0^l)/(p_1^l(e^{\mu(\theta)x}N_0) - p_1^l(e^{\mu_i^{max}x}N_0)))$, the coefficient on L_1 in the objective function is positive, meaning that it is maximized at $L_1 = p_0^l L_0/(p_0^l - \alpha^{sell}(p_1^l(e^{\mu_i^{max}x}N_0) + k_l))$ with a value of $p_0^l L_0(1-\alpha^{sell})(p_1^l(e^{\mu(\theta)x}N_0) + k_l)/(p_0^l - \alpha^{sell}(p_1^l(e^{\mu_i^{max}x}N_0) + k_l))$. This value is less than the maximized value when $(\alpha^{sell})^* = 0$ because for such α^{sell}, $(1-\alpha^{sell})p_0^l < p_0^l - \alpha^{sell}(p_1^l(e^{\mu_i^{max}x}N_0) + k_l)$. We have proved that the given choices are optimal in the first case. The proof that the choices are optimal in the second case is similar. The maximized objective equals $p_0^l L_0$. If $\alpha^{sell} > 0$, then the coefficient on L_1 in the objective is negative, leading to $L_1^* = 0$. If $\alpha^{sell} = 0$, then the coefficient on L_1 in the objective is zero, leading to any feasible choice of L_1^*. Finally, in the third case, the coefficient on L_1 in the objective is negative for all α^{sell}, leading to $L_1^* = 0$, in which case $(\alpha^{sell})^*$ does not affect the objective.

In all of the equilibrium choices we have just listed, $(\alpha^{sell})^* > 0$ only if $L_1^* = 0$ or if $p_0^l = p_1^l(e^{\mu_i^{max}x}N_0) + k_l$. In either case, $p_0^\pi = p_0^l L_1$ by Lemma IA2. □

We now use Lemmas IA1 and IA2 to formulate and prove a lemma that characterizes the equilibrium house price at $t = 0$ as $k_l \to 0$. The lemma relies on the following definitions: $\mu_i^{max} = \mu(\theta_i^{max})$ is the belief of the most optimistic equity investor, $\theta_d^{sup} = \sup\{\theta \in \operatorname{supp} f_d \mid L_0 > 0\}$ is the least upper-bound of the beliefs of developers endowed with land, and $N_0^*(x, z, f_r, \theta_d^{max})$ is the value of $N_0^*(x, z)$ in Proposition 2 given f_r and θ_d^{max}.

LEMMA IA4: *Suppose that $x, z > 0$. If $\sum_{\theta > \theta_i^{max}} L_0 = 0$, then the limit of the equilibrium* *house price at $t = 0$ as $k_l \to 0$ equals*

$$p_0^h(N_0, x, z) = \begin{cases} 2k & \text{if } N_0 \leq e^{-\mu_i^{max} x} \\ k + ke^{\mu_i^{max} x/\epsilon} N_0^{1/\epsilon} & \text{if } e^{-\mu_i^{max} x} < N_0 < N_0^*(x, z, f_r, \theta_i^{max}) \\ k(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon}) N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z, f_r, \theta_i^{max}). \end{cases}$$

If $\sum_{\theta > \theta_i^{max}} L_0 > 0$, then the limit of the equilibrium house price at $t = 0$ as $k_l \to 0$ equals

$$p_0^h(N_0, x, z) = \begin{cases} 2k & \text{if } N_0 \leq \min(e^{-\mu_i^{max} x}, N_0^{**}(x, z)) \\ k + ke^{\mu_i^{max} x/\epsilon} N_0^{1/\epsilon} & \text{if } e^{-\mu_i^{max} x} < N_0 < N_0^{**}(x, z) \\ k + ke^{\mu_d^{agg}(N_0,x,z)x/\epsilon} N_0^{1/\epsilon} & \text{if } N_0^{**}(x, z) < N_0 < N_0^*(x, z, f_r, \theta_d^{sup}) \\ k(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon}) N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z, f_r, \theta_d^{sup}). \end{cases}$$

Here $\mu_d^{agg}(N_0, x, z)$ increases in N_0 and depends on the beliefs and endowments of only those developers for whom $\theta > \theta_i^{max}$ and $L_0 > 0$, and $N_0^*(x, z, f_r, \theta_d^{sup}) \geq N_0^{**}(x, z) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ with equality if and only if $N_0^{**}(x, z) = \infty$, which occurs if and only if $\int_{\theta \geq \theta_i^{max}} f_r(\theta) d\theta = 0$ and $\int_{\theta < \theta_i^{max}} (e^{\mu_i^{max} x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon} f_r(\theta) d\theta \leq \sum_{\theta \leq \theta_i^{max}} L_0/S$.

Proof of Lemma IA4: The proof of Lemma IA3 fully characterized developer choices of end-of-period landholdings at $t = 0$ given p_0^l. The land price constitutes an equilibrium when the space demanded by potential residents given p_0^l plus the sum of L_1 across developers equals S (the proof of Lemma IA1 shows that $H_1 = 0$ for all developers). If $p_0^l = p_1^l(e^{\mu_i^{max} x} N_0) + k_l$, then the total L_1 across developers can take on any value at least $\sum_{\theta | p_1^l(e^{\mu_i^{max} x} N_0) < p_1^l(e^{\mu(\theta)x} N_0)} L_0$. Equilibrium holds in this case if and only if

$$\sum_{\theta | p_1^h(e^{\mu_i^{max} x} N_0) \geq p_1^h(e^{\mu(\theta)x} N_0)} L_0/S \geq \int_\Theta N_0 D(p_1^h(e^{\mu_i^{max} x} N_0) + k + k_l - p_1^h(e^{\mu(\theta)x} N_0)) f_r(\theta) d\theta. \quad \text{(IA1)}$$

By the same argument about the right side of (A1) in the proof of Proposition 2, the right side of (IA1) weakly and continuously increases in N_0 and $\to 0$ as $N_0 \to 0$. The left side of (IA1) equals

$$\sum_{\theta | p_1^h(e^{\mu_i^{max} x} N_0) \geq p_1^h(e^{\mu(\theta)x} N_0)} L_0/S = \begin{cases} 1 & \text{if } N_0 \leq e^{-\mu_d^{max} x} \\ 1 - \sum_{\theta | e^{\mu(\theta)x} N_0 > 1} L_0/S & \text{if } e^{-\mu_d^{max} x} \leq N_0 \leq e^{-\mu_i^{max} x} \\ 1 - \sum_{\theta > \theta_i^{max}} L_0/S & \text{if } N_0 \geq e^{-\mu_i^{max} x}, \end{cases}$$

which weakly decreases in N_0 and is left-continuous. As a result, there is $N_0^{**}(x, z, k_l) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ such that (IA1) holds if and only if $N_0 \leq N_0^{**}(x, z, k_l)$. Because the right side of (IA1) decreases in k_l, $N_0^{**}(x, z, k_l)$ increases in k_l, meaning that $N_0^{**}(x, z) \equiv \lim_{k_l \to 0} N_0^{**}(x, z, k_l)$ exists.

We pause here to prove two needed facts about $N_0^{**}(x, z, k_l)$. As a point of notation, define $N_0^*(x, z, f_r, \theta_d^{max}, k_l)$ to be the value of $N_0^*(x, z)$ obtained from (A1) with $k + k_l$ in place of k inside the integral. First: if $\sum_{\theta > \theta_i^{max}} L_0 = 0$, then the left side of (IA1) reduces to one. It follows from comparison with (A1) that $N_0^{**}(x,z,k_l) = N_0^*(x,z,f_r,\theta_i^{max},k_l)$ and $N_0^{**}(x,z) = N_0^*(x,z,f_r,\theta_i^{max})$ in this case. Second: by the same argument used in Proposition 2 to analyze (A1), the limit of the right side of (IA1) as $N_0 \to \infty$ equals ∞ if $\int_{\theta \geq \theta_i^{max}} f_r(\theta)d\theta \geq 0$ and equals $\int_{\theta<\theta_i^{max}} (e^{\mu_i^{max}x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon} f_r(\theta)d\theta$ otherwise. It follows that $N_0^{**}(x,z) = \infty$ if and only if the conditions given in Lemma IA4 hold.

In the second possible equilibrium, $p_0^l > p_1^l(e^{\mu_i^{max}x}N_0)+k_l$. In this case, the total L_1 across developers may take any value between $\sum_{\theta|p_0^l<p_1^l(e^{\mu(\theta)x}N_0)+k_l} L_0$ and $\sum_{\theta|p_0^l\leq p_1^l(e^{\mu(\theta)x}N_0)+k_l} L_0$. Equilibrium holds if potential residents' demand for space at $p_0^h = p_0^l + 2k$ equals the remaining land not held by developers, that is, if p_0^h satisfies

$$\sum_{\theta|p_0^h>p_1^h(e^{\mu(\theta)x}N_0)+k+k_l} L_0/S \leq \int_\Theta N_0 D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta)d\theta \leq \sum_{\theta|p_0^h\geq p_1^h(e^{\mu(\theta)x}N_0)+k+k_l} L_0/S. \tag{IA2}$$

Such a p_0^h exists if and only if (IA1) fails. To see this, suppose (IA1) holds. The left side of (IA2) weakly increases in p_0^h, while the middle strictly decreases for $p_0^h \geq p_1^h(e^{\mu_i^{max}x}N_0)+k+k_l$ because $\theta < 0 < \theta_i^{max}$ for a positive measure of potential residents (Assumption 4). If (IA1) holds, then the left side of (IA2) is at least the middle when $p_0^h = p_1^h(e^{\mu_i^{max}x}N_0)+k+k_l$, meaning that for larger p_0^h, the left strictly exceeds the middle in violation of (IA2). Now suppose that (IA1) fails. Then the middle of (IA2) exceeds the right side at $p_0^h = p_1^h(e^{\mu_i^{max}x}N_0)+k+k_l$. Because the middle strictly and continuously decreases to zero with $p_0^h \geq p_1^h(e^{\mu_i^{max}x}N_0)+k+k_l$, there exists a unique solution to (IA2), which we call $p_0^h(N_0,x,z,k_l)$. Existence and uniqueness follow from the fact that the greatest lower bound of the p_0^h for which the left inequality fails equals the lowest upper bound of the p_0^h for which the right inequality fails.

We further partition this possible equilibrium into two cases. Set $\mu_d^{sup} = \mu(\theta_d^{sup})$. In the first case,

$$1 \geq \int_\Theta N_0 D(p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta)d\theta. \tag{IA3}$$

At $p_0^h = p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$, the right side of (IA2) equals one. As a result, if (IA3) fails, then $p_0^h(N_0,x,z,k_l)$ satisfies (A2). If (IA3) holds, then if $p_0^h > p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$, the left and right of (IA2) equal one while the middle is less than one. As a result, $p_0^h(N_0,x,z,k_l) \leq p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$. By the same argument given in the proof of Proposition 2 concerning (A1), (IA3) holds if and only if $N_0 \leq N_0^*(x,z,f_r,\theta_d^{sup},k_l)$.

In summary, a unique equilibrium house price at $t = 0$ exists. If $N_0 \leq N_0^{**}(x,z,k_l)$, then we have $p_0^h(N_0,x,z,k_l) = p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l$. If $N_0^{**}(x,z,k_l) < N_0 < N_0^*(x,z,f_r,\theta_d^{sup},k_l)$, then $p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l < p_0^h(N_0,x,z,k_l) \leq p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$. If $N_0 > N_0^{**}(x,z,k_l)$ and $N_0 \geq N_0^*(x,z,f_r,\theta_d^{sup},k_l)$, then $p_0^h(N_0,x,z,k_l)$ satisfies (A2).

If $\sum_{\theta>\theta_i^{max}} L_0 = 0$, then $\mu_d^{sup} \leq \mu_i^{max}$. In this case, $N_0^{**}(x,z,k_l) = N_0^*(x,z,f_r,\theta_i^{max},k_l) \geq N_0^*(x,z,f_r,\theta_d^{sup},k_l)$, where the equality was proved earlier and the inequality follows because N_0^* increases in its fourth argument. As a result, the equilibrium house price in this case equals

$$p_0^h(N_0,x,z,k_l) = \begin{cases} 2k+k_l & \text{if } N_0 \le e^{-\mu_i^{max}x} \\ k+ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon}+k_l & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^*(x,z,f_r,\theta_i^{max},k_l) \\ k(1+e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \ge N_0^*(x,z,f_r,\theta_i^{max},k_l). \end{cases}$$

Taking the limit as $k_l \to 0$ yields the formula in Lemma IA4.

If $\sum_{\theta>\theta_i^{max}} L_0 > 0$, then $\mu_d^{sup} > \mu_i^{max}$. From comparing (IA1) to (IA3), we see that $N_0^{**}(x,z,k_l) \le N_0^*(x,z,f_r,\theta_d^{sup},k_l)$ and $N_0^{**}(x,z) \le N_0^*(x,z,f_r,\theta_d^{sup})$, with equality in each if and only if the respective left side equals ∞. If $N_0^{**}(x,z,k_l) < N_0 \le N_0^*(x,z,f_r,\theta_d^{sup},k_l)$, then $p_1^h(e^{\mu_i^{max}x}N_0)+k+k_l < p_0^h(N_0,x,z,k_l) \le p_1^h(e^{\mu_d^{sup}x}N_0)+k+k_l$. Over this range, the only developers on which (IA2) depends are those with positive land holdings and beliefs in $\{\theta \mid p_1^h(e^{\mu_i^{max}x}N_0) < p_1^h(e^{\mu(\theta)x}N_0)\} = \{\theta \mid \theta > \theta_i^{max}\}$. It follows that $p_0^h(N_0,x,z,k_l)$ in this range depends on only these developers. Because $p_1^h(e^{\mu_d^{sup}x}N_0) = ke^{\mu_d^{sup}x/\epsilon}N_0^{1/\epsilon}$ in this range, there exists a unique $\mu_d^{agg}(N_0,x,z,k_l) \in [-\log(N_0)/x, \mu_d^{sup}]$ such that on this range of N_0, $p_0^h(N_0,x,z,k_l) = k+ke^{\mu_d^{agg}(N_0,x,z,k_l)x/\epsilon}N_0^{1/\epsilon}+k_l$. Because $p_0^h(N_0,x,z,k_l)$ increases in k_1 and is bounded on this range, $\lim_{k_l\to 0}\mu_d^{agg}(N_0,x,z,k_l)$ exists; we denote it by $\mu_d^{agg}(N_0,x,z)$. The middle of (IA2) increases in N_0, as shown in the the proof of Proposition 2, so $\mu_d^{agg}(N_0,x,z)$ increases in N_0. Putting everything together, we have that

$$p_0^h(N_0,x,z,k_l) =$$
$$\begin{cases} 2k+k_l & \text{if } N_0 \le \min(e^{-\mu_i^{max}x}, N_0^{**}(x,z,k_l)) \\ k+ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon}+k_l & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^{**}(x,z,k_l) \\ k+ke^{\mu_d^{agg}(N_0,x,z,k_l)x/\epsilon}N_0^{1/\epsilon}+k_l & \text{if } N_0^{**}(x,z,k_l) < N_0 < N_0^*(x,z,f_r,\theta_d^{sup},k_l) \\ k(1+e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \ge N_0^*(x,z,f_r,\theta_d^{sup},k_l). \end{cases}$$

when $\sum_{\theta>\theta_i^{max}} L_0 > 0$. Taking the limit as $k_l \to 0$ yields the formula in Lemma IA4.

The only point at which we use $k_l > 0$ above is for the existence of equilibrium when $p_0^h(N_0,x,z,k_l) = 2k+k_l$. In this case, $p_0^l(N_0,x,z,k_l) = k_l$, but we showed earlier that $p_0^l = 0$ can never be an equilibrium. This equilibrium exists only as a limit as $k_l \to 0$. □

The case in which $\sum_{\theta>\theta_i^{max}} L_0 > 0$ and $N_0^{**}(x,z) < N_0 < N_0^*(x,z,f_r,\theta_d^{sup})$ deserves some explanation, as the equilibrium house price in this region looks quite different than any of the prices in Proposition 2. This case occurs when demand from potential residents is at least equal to the space held by developers for whom $\theta \le \theta_i^{max}$, but is not as large as the entire space S. In such an equilibrium, developers for whom $\theta > \theta_i^{max}$ become the marginal owners of space and hold some land in equilibrium. The equilibrium house price aggregates the beliefs of such landowning developers through μ_d^{agg}. This case always occurs unless demand from potential residents when the optimistic equity investors price space is never large enough to cut into the landholdings of these very optimistic developers; this condition is precisely the one at the end of Lemma IA4.

Finally, we build on the proof of Lemma IA4 to prove Proposition 7.

Proof of Proposition 7: The claim that the equilibrium house price equals $p_0^h(N_0,x,z,f_r,f_i)$

when $\sum_{\theta > \theta_i^{max}} L_0 = 0$ follows immediately from comparing the pricing formula in Lemma IA4 to that in Proposition 2.

To prove the remaining claims, we first solve for the optimal equity purchases for investors. By Lemma IA2, the objective function for an equity investor with respect to a given developer is to maximize $\alpha^{buy}(p_1^l(e^{\mu(\theta)x}N_0)+k_l)L_1 - \max(\alpha^{buy}, (1-k_s)\alpha^{buy})(p_1^l(e^{\mu_i^{max}x}N_0)+k_l)L_1$ subject to $\alpha^{buy} \geq -\overline{\alpha}$. If $L_1 > 0$, then the optimal choice for the equity investor is

$$
\begin{array}{ll}
(\alpha^{buy})^* \geq 0 & \text{if } p_1^l(e^{\mu(\theta)x}N_0) = p_1^l(e^{\mu_i^{max}x}N_0) \\
(\alpha^{buy})^* = 0 & \text{if } p_1^l(e^{\mu(\theta)x}N_0) \in \big((1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_s k_l, p_1^l(e^{\mu_i^{max}x}N_0)\big) \\
(\alpha^{buy})^* \in [-\overline{\alpha}, 0] & \text{if } p_1^l(e^{\mu(\theta)x}N_0) = (1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_s k_l \\
(\alpha^{buy})^* = -\overline{\alpha} & \text{if } p_1^l(e^{\mu(\theta)x}N_0) < (1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_s k_l.
\end{array}
$$

When $xz = 0$, $p_1^l(e^{\mu(\theta)x}N_0) = p_1^l(e^{\mu_i^{max}x}N_0) > (1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_s k_l$ because $k_s > 0$, so $(\alpha^{buy})^* \geq 0$ for all equity investors and developers for whom $L_1 > 0$. The claim that the aggregate value of short claims equals zero when $xz = 0$ is proved. For the second claim about the $xz = 0$ case, first consider the possibility that $p_0^l > p_1^l(e^{\overline{\mu}x}N_0) + k_l$. Then the proof of Lemma IA4 shows that $L_1^* = 0$ for all developers and that $(\alpha^{sell})^* = 0$ is possible for all developers, meaning that an equilibrium exists in which no equity issuance occurs and in which $(H_0^{build})^* = L_0$ and $(L_0^{buy})^* = 0$ for all developers. Now consider the other possibility, that $p_0^l = p_1^l(e^{\overline{\mu}x}N_0) + k_l$. Then by the proof of Lemma IA4, each developer may choose $(\alpha^{sell})^* = 0$ and $L_1^* \leq L_0$. As a result, no equity is issued, and the sum of L_1^* across developers can take on any value between zero and S, meaning that we may find an equilibrium in which $(L_0^{buy})^* = 0$ for all developers and $(H_0^{build})^*$ is chosen to clear the housing market.

We turn now to the remaining claims about the $xz > 0$ case. We define $N_0^{***}(x, z, k_l)$ to be the least upper bound of N_0 such that

$$
\sum_{\substack{\theta < \theta_i^{max} \\ \text{developers w/o} \\ \text{access to equity}}} L_0/S > \int_\Theta N_0 D(p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta) d\theta. \tag{IA4}
$$

As discussed in the proof of Lemma IA4, the right side of (IA4) continuously increases in N_0 and approaches zero as $N_0 \to 0$, so $N_0^{***}(x, z, k_l) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ exists. Because the right side of (IA4) is continuous in k_l, we may define $N_0^{***}(x, z) = \lim_{k_l \to 0} N_0^{***}(x, z, k_l) = N_0^{***}(x, z, 0)$. Furthermore, substituting $N_0 = e^{-\mu_i^{max}x}$ into the right side of (IA4) when $k_l = 0$ yields $e^{-\mu_i^{max}x}$, so because the left side exceeds $e^{-\mu_i^{max}x}$, we must have $N_0^{***}(x, z, k_l) \geq N_0^{***}(x, z) > e^{-\mu_i^{max}x}$ (N_0^{***} decreases in k_l). The left side of (IA4) is less than or equal to the left side of (IA1) as shown in the analysis after (IA1), so $N_0^{***}(x, z, k_l) \leq N_0^{**}(x, z, k_l)$ and $N_0^{***}(x, z) \leq N_0^{**}(x, z)$.

We prove the remaining claims about the $xz > 0$ case for N_0 such that $e^{-\mu_i^{max}x} < N_0 < N_0^{***}(x, z)$. Such N_0 satisfy $e^{-\mu_i^{max}x} < N_0 < N_0^{**}(x, z, k_l)$ given the inequalities above. By the proof of Lemma IA4, $p_0^l = p_1^l(e^{\mu_i^{max}x}N_0) + k_l$ in equilibrium for such N_0. Assume for a contradiction that $(\alpha^{sell})^* L_1^* = 0$ for all developers. The largest possible sum of L_1^* across all developers equals $\sum_{\theta \geq \theta_i^{max}} L_0$. An equilibrium is possible only if the housing demand from potential residents is at least equal to the remaining land. This condition is

$$\sum_{\theta<\theta_i^{max}} L_0/S \leq \int_\Theta N_0 D(p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta) d\theta,$$

which fails for $N_0 < N_0^{***}(x,z)$ due to (IA4), providing the necessary contradiction and proving that the aggregate value of issued equity is positive.

From one of the developer constraints, $(L_0^{buy})^* - (H_0^{build})^* = L_1^* - L_0$, so the sum of the former across equity-issuing developers equals the sum of the latter across them. Assume for a contradiction that the latter sum is ≤ 0. For all developers not issuing equity, $L_1^* \leq L_0$, with $L_1^* = 0$ for developers without access to the equity market for whom $\theta < \theta_i^{max}$. As a result, the total demand for space may equal S only if the precise opposite of (IA4) holds. Because $N_0 < N_0^{***}(x,z)$, we have a contradiction that proves that developers who issue equity in the aggregate buy land beyond construction needs.

We now prove the statement about shorting of equity-issuing developers. Pick any $\theta' < \theta_i^{max}$ such that $\int_{\theta\leq\theta'} f_i(\theta)d\theta > 0$, where f_i is the distribution of θ across equity investors (Assumption 4 guarantees the existence of θ'). We will show that we can find k_s small enough that $(\alpha^{buy})^* = -\overline{\alpha}$ for all $\theta \leq \theta'$. If $e^{\mu(\theta)x}N_0 \leq 1$, then $(\alpha^{buy})^* = -\overline{\alpha}$ if and only if $k < (1-k_s)ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} - k_s k_l$. As $k_s \to 0$, the right side approaches something greater than k, so we can find $k_s > 0$ small enough that $(\alpha^{buy})^* = -\overline{\alpha}$ for all θ with $e^{\mu(\theta)x}N_0 \leq 1$. Now consider $\theta \leq \theta'$ with $e^{\mu(\theta)x}N_0 > 1$. An equity investor with such θ sets $(\alpha^{buy})^* = -\overline{\alpha}$ if and only if $ke^{\mu(\theta)x/\epsilon}N_0^{1/\epsilon} < (1-k_s)ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} - k_s k_l$. This equation holds if $ke^{\mu(\theta')x/\epsilon}N_0^{1/\epsilon} < (1-k_s)ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} - k_s k_l$. Because $\theta' < \theta_i^{max}$, the right side approaches something greater than the left as $k_s \to 0$, so we may choose k_s small enough that the inequality holds. We may pick k_s small enough that $(\alpha^{buy})^* = -\overline{\alpha}$ for all $\theta \leq \theta'$, as desired.

From Lemma IA2, the price of the claim on a developer for whom $(\alpha^{sell})^* > 0$ and $L_1^* > 0$ equals $p_0^\pi = (ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} + k_l)L_1^*$. This expression increases strictly in x, as claimed. The price at $t = 1$ equals $p_1^\pi = (p_1^l(e^{\mu^{true}x}N_0) + k_l)L_1^*$, which is strictly less than p_0^π if and only if $\mu^{true} < \mu_i^{max}$.

□

II. Rental Extension

A share $\chi \in [0,1)$ of residents are of type $a = 1$ and get flow utility only from renting; the remainder are of type $a = 0$ and get flow utility only from owning.[2] The type a is distributed independently from v and θ. All residents can act as landlords, but developers cannot (the developer problem remains the same as before). We denote by R_t^{buy} the quantity of housing rented as a tenant and by R_t^{sell} the quantity rented as a landlord. The rental price of housing

[2]We rule out $\chi = 1$ because f_r^χ does not satisfy Assumption 4 when $\chi = 1$, meaning that the expressions $p_0^h(N_0, x, z, f_r^\chi, f_d)$ and $N_0^*(x, z, f_r^\chi)$ that appear in Proposition 8 are not well-defined. The existence of equilibrium does not depend on $\chi \neq 1$, so by continuity the $\chi = 1$ equilibrium equals the limiting equilibrium as $\chi \to 1$.

equals p_t^r. At $t = 1$, an arriving potential resident chooses $(H_1^{buy})^*, (R_1^{buy})^*$, and $(R_1^{sell})^*$ from

$$\begin{aligned} \underset{H_1^{buy}, R_1^{buy}, R_1^{sell}}{\arg\max} \quad & \Big(a\iota(R_1^{buy}) + (1-a)\iota(H_1^{buy} - R_1^{sell})\Big) v - p_1^h H_1^{buy} - p_1^r (R_1^{buy} - R_1^{sell}) \\ \text{subject to} \quad & 0 \leq H_1^{buy} \\ & 0 \leq R_1^{buy} \\ & 0 \leq R_1^{sell} \\ & R_1^{sell} \leq H_1^{buy}, \end{aligned}$$

where $\iota(R) = 1$ if $R \geq 1$ and 0 otherwise. The utility $u(p_1^h, B_1, v, a, H_0^{buy}, R_0^{buy}, R_0^{sell})$ at $t = 1$ of a potential resident of type a and v who arrived at $t = 0$ and chose H_0^{buy}, R_0^{buy}, and R_0^{sell} equals

$$\begin{aligned} \underset{H_1^{sell}}{\arg\max} \quad & \Big(a\iota(R_0^{buy}) + (1-a)\iota(H_0^{buy} - R_0^{sell})\Big) v + H_1^{sell} p_1^h + B_1 \\ \text{subject to} \quad & 0 \leq H_1^{sell} \\ & H_1^{sell} \leq H_0^{buy}. \end{aligned}$$

At $t = 0$, arriving potential residents maximize the subjective expectation of their utility by choosing $(H_0^{buy})^*$, $(R_0^{buy})^*$, and $(R_0^{sell})^*$ from

$$\begin{aligned} \underset{H_0^{buy}, R_0^{buy}, R_0^{sell}}{\arg\max} \quad & \mathrm{E}u(p_1^h, B_1, v, a, H_0^{buy}, R_0^{buy}, R_0^{sell}) \\ \text{subject to} \quad & 0 \leq H_0^{buy} \\ & 0 \leq R_0^{buy} \\ & 0 \leq R_0^{sell} \\ & R_0^{sell} \leq H_0^{buy} \\ & B_0 = -p_0^h H_0^{buy} - p_0^r (R_0^{buy} - R_0^{sell}). \end{aligned}$$

Equilibrium is the same as before with the addition of the condition that the sum of $(R_t^{buy})^*$ across all residents equals the sum of $(R_t^{sell})^*$ across them at each t. The following lemma characterizes this equilibrium at $t = 1$.

LEMMA IA5: *A unique equilibrium at $t = 1$ exists and coincides with that given by Lemma 1.*

Proof: A potential resident arriving at $t = 1$ of type $a = 1$ gets utility $v - p_1^r$ from setting $R_1^{buy} = 1$ and utility 0 from setting $R_1^{buy} = 0$ (all other choices are dominated). The sum of $(R_1^{buy})^*$ therefore equals $\chi N_1 SD(p_1^r)$.

Increasing H_1^{buy} and R_1^{sell} by the same amount increases utility if $p_1^r > p_1^h$ and decreases utility if $p_1^r < p_1^h$. The former cannot hold in equilibrium, as it leads to unlimited housing demand, which cannot be matched by the limited supply. The latter cannot hold in equilibrium if $\chi > 0$, as it would lead to zero rental supply, which cannot be matched by rental demand, which is positive if $\chi > 0$. If $\chi = 0$, $p_1^r < p_1^h$ can hold in equilibrium if $(R_0^{sell})^* = 0$

for all potential residents. Therefore, $p_1^h = p_1^r$ or $\chi = 0$ and $p_1^r < p_1^h$.

If $\chi > 0$, then a potential resident arriving at $t = 1$ of type $a = 1$ sets $(H_1^{buy})^* = (R_1^{sell})^*$. Arriving potential residents of type $a = 0$ set $(R_1^{buy})^* = 0$, because $p_1^r = p_1^h \geq k > 0$ in the case in which $\chi > 0$, or because clearing of the rental market in the case in which $\chi = 0$ and $p_1^r < p_1^h$ requires it (the equilibrium possibilities are then $p_1^r \in [0, p_1^h)$). Setting $R_1^{sell} = 0$ in the case in which $\chi = 0$ and $p_1^r < p_1^h$, arriving potential residents of type $a = 0$ get utility $v - p_1^h$ if $H_1^{buy} - R_1^{sell} = 1$ and utility 0 if $H_1^{buy} - R_1^{sell} = 0$ (all other choices of $H_1^{buy} - R_1^{sell}$ are dominated). The total of $(H_1^{buy})^* - (R_1^{sell})^*$ across these potential residents equals $(1-\chi)N_1 SD(p_1^h)$.

The total of $(H_1^{buy})^* - (R_1^{sell})^* + (R_1^{buy})^*$ across all residents equals $N_1 SD(p_1^h)$. Because the rental market clears, the total of $(H_1^{buy})^*$ equals $N_1 SD(p_1^h)$, which coincides with housing demand in the model of Section I. As shown in the proof of Lemma 1, the sum of $(H_1^{sell})^*$ across departing residents is irrelevant for equilibrium prices at $t = 1$, so we are done. $\square$

We now prove Proposition 8.

Proof of Proposition 8: For clarity, we divide the proof into three parts.

Part 1: Equilibrium house price at $t = 0$

Consider potential residents for whom $a = 1$. If $R_0^{buy} \in [0, 1)$, then utility is $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$. If $p_0^r < 0$, then R_0^{buy} cannot be chosen to maximize utility, so $p_0^r \geq 0$ in equilibrium. As a result, utility weakly increases in R_0^{sell}, so it is maximized when $R_0^{sell} = H_0^{buy}$ and $R_0^{buy} = 0$ at $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. If $R_0^{buy} \geq 1$, then utility equals $v + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$, which is maximized when $R_0^{buy} = 1$ and $R_0^{sell} = H_0^{buy}$ at $v - p_0^r + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. Thus, unless $p_0^r = 0$ (which we consider below), the sum of $(R_0^{buy})^*$ across potential residents of type $a = 1$ equals $\chi N_0 SD(p_0^r)$, and the sum of $(R_0^{sell})^*$ across them equals the sum of $(H_0^{buy})^*$ across them.

Consider the problem for potential residents with $a = 0$. If $H_0^{buy} - R_0^{sell} \in [0, 1)$, then utility equals $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$. Utility is maximized when $R_0^{sell} = H_0^{buy}$ at $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. If $p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h > 0$ for any $\theta \in \operatorname{supp} f_r$, then utility cannot be maximized. As a result, $p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h \leq 0$ for all $\theta \in \operatorname{supp} f_r$, and utility is maximized at zero. If $H_0^{buy} - R_0^{sell} \geq 1$, then utility equals $v + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$, which weakly rises in R_0^{sell}. Utility is maximized when $R_0^{sell} = H_0^{buy} - 1$ at $v - p_0^r + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. Because $p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h \leq 0$ for all $\theta \in \operatorname{supp} f_r$, utility is maximized when $R_0^{sell} = 0$ and $H_0^{buy} = 1$ at $v - p_0^h + p_1^h(e^{\mu(\theta)x}N_0)$. Thus, unless $p_0^r = 0$, the sum of $(R_0^{buy})^*$ across these potential residents equals zero, and the sum of $(H_0^{buy})^*$ across them exceeds the sum of $(R_0^{sell})^*$ across then by $(1-\chi)N_0 S \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta$.

Combining the two cases, we see that if $p_0^r > 0$, the sum of $(H_0^{buy})^*$ across potential residents equals $\chi N_0 SD(p_0^r) + (1-\chi)N_0 S \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta$ due to the clearing of the rental market.

If $p_1^h(e^{\mu_r^{max}x}N_0) + p_0^r - p_0^h < 0$, then $(R_0^{sell})^* = 0$ for all potential residents. The rental market can clear only if $\chi N_0 SD(p_0^r) = 0$, which can hold only if $\chi = 0$. In this case, rental supply and demand equals zero for all potential residents (or $p_0^r = 0$), in which case the rental market becomes irrelevant and the equilibrium reduces to that analyzed in Section III. Therefore, for the rest of the proof we assume that $\chi > 0$. In this case, $(R_0^{sell})^* > 0$ for some potential residents, so $p_1^h(e^{\mu_r^{max}x}N_0) + p_0^r - p_0^h = 0$.

As shown in the proof of Lemma 2, either $p_0^h = k + p_1^h(e^{\mu_d^{max}x}N_0)$, in which case developers for whom $p_1^h(e^{\mu(\theta)x}N_0) = p_1^h(e^{\mu_d^{max}x}N_0)$ may choose any $L_1 \geq 0$, or $p_0^h > k + p_1^h(e^{\mu_d^{max}x}N_0)$, in which case $L_1 = 0$ for all developers. The former may hold in equilibrium if and only if the resulting housing demand falls short of S:

$$\begin{aligned} 1 \geq\ & \chi N_0 D(p_1^h(e^{\mu_d^{max}x}N_0) - p_1^h(e^{\mu_r^{max}x}N_0)) \\ & + (1-\chi)N_0 \int_\Theta D(p_1^h(e^{\mu_d^{max}x}N_0) - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta. \end{aligned} \tag{IA5}$$

This inequality is the same as (A1) but with f_r replaced by f_r^χ, so (IA5) holds if and only if $N_0 \leq N_0^*(x,z,f_r^\chi)$. For such N_0, $p_0^h = k + p_1^h(e^{\mu_d^{max}x}N_0) = p_0^h(N_0,x,z,f_r^\chi,f_d)$. If $N_0 > N_0^*(x,z,f_r^\chi)$, then p_0^h must equate total housing demand with S, meaning that it is the unique value satisfying

$$1 = \chi N_0 D(p_0^h - p_1^h(e^{\mu_r^{max}x}N_0)) + (1-\chi)N_0 \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta. \tag{IA6}$$

This equation coincides with (A2) but with f_r^χ in place of f_r, so the equilibrium house price at $t=0$ equals $p_0^h(N_0,x,z,f_r^\chi,f_d)$ for $N_0 \geq N_0^*(x,z,f_r^\chi)$.

Part 2: Nonmonotonicity of house price boom

According to Proposition 5, the boom is strictly maximized at $N_0 = 1$ if Assumption 5 holds when applied to f_r^χ in place of f_r. The first condition in the assumption, $e^{\mu_d^{max}x/\epsilon} > e^{\mu_r^{max}x/\epsilon} - 1$, continues to hold because the maxima of f_r and f_r^χ coincide. The second condition applied to f_r^χ is

$$1 > \chi\left(1 + e^{\mu_d^{max}x/\epsilon} - e^{\mu_r^{max}/\epsilon}\right)^{-\epsilon} + (1-\chi)\int_\Theta \left(1 + e^{\mu_d^{max}x/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon} f_r(\theta)d\theta. \tag{IA7}$$

This inequality holds for $\chi = 0$ by assumption. The right side of (IA7) increases continuously in χ because $\mu_r^{max} > \mu(\theta)$ for all $\theta < \theta_r^{max}$, so (IA7) holds for all χ if and only if it holds for $\chi = 1$. When $\chi = 1$, (IA7) reduces to $\mu_r^{max} < \mu_d^{max}$. If $\mu_r^{max} \geq \mu_d^{max}$, then by the intermediate value theorem there exists $\chi^*(x,z) \in (0,1]$ such that (IA7) holds if $\chi < \chi^*(x,z)$. When $\mu_r^{max} = \mu_d^{max}$, (IA7) holds as an equality, so $\chi^*(x,z) = 1$.

Part 3: House price boom and rental share

As shown earlier in this proof,

$$\frac{\sum (R_0^{buy})^*}{\sum (H_0^{buy})^*} = \frac{\chi D(p_0^h(N_0,x,z,\chi) - p_1^h(e^{\mu_r^{max}x}N_0))}{\int_\Theta D(p_0^h(N_0,x,z,\chi) - p_1^h(e^{\mu(\theta)x}N_0))f_r^\chi(\theta)d\theta}.$$

If $xz = 0$, then $\mu(\theta) = \mu_r^{max}$ for all θ, so this fraction equals χ.

We now fix a value of N_0 and consider the case in which $xz > 0$. The right side of (IA5) weakly increases in χ, so $N_0^*(x,z,f_r^\chi)$ weakly and continuously decreases in χ. As a result, if $N_0 < N_0^*(x,z,\chi)$, then a marginal increase in χ has no bearing on $p_0^h(N_0,x,z,f_r^\chi,f_d)$, as this equilibrium price is independent of χ for $N_0 < N_0^*(x,z,f_r^\chi)$. If $N_0 \geq N_0^*(x,z,f_r^\chi)$, then $p_0^h(N_0,x,z,f_r^\chi,f_d)$ solves (IA6). Because $N_0 \geq N_0^*(x,z,f_r^\chi) > 1$, the integral in (IA6) evaluated at $p_0^h = p_0^h(N_0, x, z, f_r^\chi, f_d)$ must be less than one. It follows that $D(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max}x}N_0)) > \int_\Theta D(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta$, so an increase in χ increases the right side of (IA6) holding $p_0^h = p_0^h(N_0, x, z, f_r^\chi, f_d)$ constant. Because the right side of (IA6) weakly decreases in p_0^h, it follows that $p_0^h(N_0, x, z, f_r^\chi, f_d)$ strictly increases in χ, as desired.

$\square$

III. Supply Elasticity Extension

Developers may rent out undeveloped land on spot markets each period to firms, such as banana stands, that use the city's land as an input. We denote the land rent by r_t^l. Spot land demand of firms equals $SD^l(r_t^l)$, where D^l satisfies the following.

ASSUMPTION IA1: $D^l : \mathbb{R}_+ \to \mathbb{R}_+$ *is continuously differentiable and decreases,* $-r(D^l)'(r)/D^l(r)$ *weakly decreases, and* $\lim_{r\to 0} D^l(r) \geq 1 > \lim_{r\to\infty} D^l(r)$.

The positivity of D^l guarantees that some vacant land exists in equilibrium, a property that makes analyzing the equilibrium easier. The condition on $r(D^l)'/D^l$ means that land demand becomes weakly less elastic as its spot price rises, so that it is weakly costlier to use each marginal unit of land. The first limit implies that land demand is at least equal to available space when land is free and leads to a positive spot price in equilibrium. The second limit implies that land demand falls below available space at a high enough price and leads to the existence of equilibrium.

Each developer chooses the quantity L_t^{rent} of land to rent on the spot market. At $t = 1$, the liquidation value π of a developer is the outcome of the constrained optimization problem

$$\begin{aligned} \pi(p_1^h, p_1^l, r_1^l, H_1, L_1, B_1) = \max_{H_1^{sell}, L_1^{buy}, H_1^{build}, L_1^{rent}} \quad & p_1^h H_1^{sell} - p_1^l L_1^{buy} - kH_1^{build} + r_1^l L_1^{rent} + B_1 \\ \text{subject to} \quad & H_1^{sell} \leq H_1 + H_1^{build} \\ & H_1^{build} \leq L_1 + L_1^{buy} - L_1^{rent}. \end{aligned}$$

The actions $(H_1^{sell})^*$, $(L_1^{buy})^*$, $(H_1^{build})^*$, and $(L_1^{rent})^*$ chosen by the developer maximize this problem. At $t = 0$ each developer chooses $(H_0^{sell})^*$, $(L_0^{buy})^*$, $(H_0^{build})^*$, and $(L_0^{rent})^*$ from

$$\begin{aligned} \arg\max_{H_0^{sell}, L_0^{buy}, H_0^{build}, L_0^{rent}} \quad & \mathrm{E}\pi(p_1^h, p_1^l, r_1^l, H_1, L_1, B_1) \\ \text{subject to} \quad & H_0^{sell} \leq H_0^{build} \\ & H_0^{build} \leq L_0 + L_0^{buy} - L_0^{rent} \\ & H_1 = H_0^{build} - H_0^{sell} \\ & L_1 = L_0 + L_0^{buy} - H_0^{build} \\ & B_1 = p_0^h H_0^{sell} - p_0^l L_0^{buy} - 2kH_0^{build} + r_0^l L_0^{rent}. \end{aligned}$$

The potential resident problems are the same as in Section II (of the Internet Appendix). Equilibrium is the same as before with the addition of the condition that the sum of $(L_t^{rent})^*$ across developers equals $SD^l(r_t^l)$ at each t. The following lemma characterizes this equilibrium at $t = 1$.

LEMMA IA6: *Given N_1, a unique equilibrium at $t = 1$ exists. In this equilibrium, $p_1^h - k = p_1^l = r_1^l > 0$.*

Proof: If $r_1^l \neq p_1^l$, then developers cannot maximize π because holding $L_1^{buy} - L_1^{rent}$ constant and increasing L_1^{buy} and L_1^{rent} always increases π if $r_1^l > p_1^l$ and decreases π if $r_1^l < p_1^l$. So $p_1^l = r_1^l$ in any equilibrium. For the same reasons given in the proof of Lemma 1, $p_1^h = p_1^l + k$ in any equilibrium.

From the proof of Lemma IA5, housing demand from arriving potential residents at $t = 1$ equals $SN_1D(p_1^h)$. Land demand from firms equals $SD^l(r_1^l)$. If $r_1^l \leq 0$, then demand for space is either not defined or exceeds S. Therefore, in any equilibrium $r_1^l > 0$. It follows that $(L_1^{rent})^* + (H_1^{build})^* = L_1 + (L_1^{buy})^*$ for all developers. Similarly, $(H_1^{sell})^* = H_1 + (H_1^{build})^*$ for all developers. Therefore, the sum of $(H_1^{sell})^* + (L_1^{rent})^*$ across developers equals the sum of $H_1 + L_1$ across them. All space other than $H_1 + L_1$ is owned by departing potential residents at the beginning of $t = 1$. The clearing of the land spot market and the housing market therefore imply that in equilibrium, $1 = D^l(r_1^l) + N_1D(p_1^h)$. Substituting $r_1^l = p_1^h - k$ yields

$$1 = D^l(p_1^h - k) + N_1D(p_1^h). \tag{IA8}$$

Because $r_1^l > 0$, $p_1^h > k$, so both D^l and D strictly decrease for possible p_1^h. The right side exceeds one as $p_1^h \to k$. As $p_1^h \to \infty$, the right side approaches something less than one by Assumption IA1. It follows that a unique value of p_1^h satisfies this equation. □

We denote equilibrium prices by $p_1^h(N_1)$, $p_1^l(N_1)$, and $r_1^l(N_1)$. The first two should not be confused with the functions defined after Lemma 1 that use the same notation.

The next lemma establishes the existence of a unique equilibrium at $t = 0$.

LEMMA IA7: *Given N_0, x, z, and χ, a unique equilibrium at $t = 0$ exists.*

Proof: For the same reasons given in the proof of Lemma IA6, $r_0^l > 0$ in any equilibrium. In the equilibrium at $t = 1$, $\pi = p_1^h H_1 + p_1^l L_1 + B_1$. By making substitutions using the constraints of the $t = 0$ developer problem, we see that the objective at $t = 0$ is to choose $H_1, L_1 \geq 0$ to maximize $(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h)H_1 + (p_1^h(e^{\mu(\theta)x}N_0) - p_0^h + k + r_0^l)L_1 + p_0^l L_0$ and that $(L_0^{rent})^* = L_1$ for each developer. Because $k + r_0^l > 0$, it follows that $H_1 = 0$ for all developers. If $p_1^h(e^{\mu(\theta)x}N_0) - p_0^h + k + r_0^l < 0$ for all developers, then $L_1 = 0$ for all of them, but then $(L_0^{rent})^* = 0$, leading to a failure of market-clearing in the land spot market because $D^l(r_0^l) > 0$. If $p_1^h(e^{\mu(\theta)x}N_0) - p_0^h + k + r_0^l > 0$ for any developer, then the objective function cannot be maximized. It follows that $p_0^h = p_1^h(e^{\mu_d^{max}x}N_0) + k + r_0^l$. Market-clearing in all markets implies that spot land demand plus total housing demand from arriving potential residents equals S. Using the equations for housing demand from the proof of Proposition 8, we form the equilibrium condition

$$\begin{aligned} 1 = {} & D^l(p_0^h - p_1^h(e^{\mu_d^{max}x}N_0) - k) + \chi N_0 D(p_0^h - p_1^h(e^{\mu_r^{max}x}N_0)) \\ & + (1-\chi)N_0 \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta. \end{aligned} \tag{IA9}$$

The right side strictly decreases in p_0^h wherever it is defined. It is defined for $p_0^h > k + p_1^h(e^{\mu_d^{max} x} N_0)$. As p_0^h approaches this value, D^l is at least one, whereas the remainder of the right side is positive. It follows that the entire right side exceeds one in the limit. As $p_0^h \to \infty$, the terms involving D go to zero, and the term involving D^l approaches something less than one according to Assumption IA1. It follows that a unique solution exists to this equation. □

We denote this unique equilibrium price by $p_0^h(N_0, x, z, \chi)$, which should not be confused with the equilibrium price given by Proposition 8.

We turn now to defining the elasticity of housing supply. The proof of Lemma IA5 shows that $r_1^h = p_1^h(N_1)$ is the unique equilibrium rent when $\chi > 0$ and is an equilibrium rent when $\chi = 0$. We define $r_1^h(N_1) = p_1^h(N_1)$. Because the housing stock at $t = 1$ equals $S - SD^l(p_1^h - k)$, the elasticity of housing supply at $t = 1$ is

$$\epsilon_1^s(N_1) = -\frac{r_1^h(N_1)(D^l)'(p_1^h(N_1) - k)}{1 - D^l(p_1^h(N_1) - k)}.$$

Similarly, the proof of Proposition 8 shows that $r_0^h = p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max} x} N_0)$ is the unique equilibrium rent when $\chi > 0$ and is an equilibrium rent when $\chi = 0$. We define $r_0^h(N_0, x, z, \chi) = p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max} x} N_0)$. Because the housing stock equals $S - SD^l(p_0^h - p_1^h(e^{\mu_d^{max} x} N_0) - k)$ at $t = 0$, the elasticity of housing supply at $t = 0$ equals

$$\epsilon_0^s(N_0, x, z, \chi) = -\frac{r_0^h(N_0, x, z, \chi)(D^l)'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_d^{max} x} N_0) - k)}{1 - D^l(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_d^{max} x} N_0) - k)}.$$

The next lemma characterizes these elasticities.

LEMMA IA8: *There exists a continuous, decreasing function $\epsilon^s : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\epsilon_0^s(N_0, x, 0, \chi) = \epsilon^s(N_0)$ and $\epsilon_1^s(N_1) = \epsilon^s(N_1)$.*

Proof of Lemma IA8: We define the function $\epsilon^s(\cdot)$ by

$$\epsilon^s(N) \equiv -\frac{p_1^h(N)(D^l)'(p_1^h(N) - k)}{1 - D^l(p_1^h(N) - k)}. \tag{IA10}$$

Given (IA8), the denominator equals $ND(p_1^h(N)) > 0$. As shown by Lemma IA6, $p_1^h(N) > k$, so the numerator is negative and well defined. It follows that $\epsilon^s(N) > 0$ for all $N > 0$. Because $p_1^h(N) > k$, the implicit function theorem applied to (IA8) implies that $p_1^h(\cdot)$ is continuous; Assumption IA1 then implies that $\epsilon^s(\cdot)$ is continuous. To show that ϵ^s decreases, we rewrite (IA10) as

$$\epsilon^s(N) = \frac{-(p_1^h(N) - k)(D^l)'(p_1^h(N) - k)}{D^l(p_1^h(N) - k)} \frac{p_1^h(N)}{p_1^h(N) - k} \frac{D^l(p_1^h(N) - k)}{1 - D^l(p_1^h(N) - k)}. \tag{IA11}$$

It is clear from (IA8) that $p_1^h(N)$ strictly increases in N because D^l and D both strictly decrease over the domains relevant in that equation. It follows that each fraction on the right of (IA11) strictly decreases in N, with the result about the first fraction following from Assumption IA1. Because $r_1^h(N_1) = p_1^h(N_1)$, $\epsilon_1^s = \epsilon^s(N_1)$. When $z = 0$, it is clear from (IA8) that $p_0^h = p_1^h(N_0) + p_1^h(e^{\overline{\mu}x}N_0)$ solves (IA9). Therefore, $r_0^h(N_0, x, 0, \chi) = p_1^h(N_0)$ and $p_0^h(N_0, x, 0, \chi) - p_1^h(e^{\mu_x^{max}}N_0) = p_1^h(N_0)$ when $z = 0$. It follows that $\epsilon_0^s(N_0, x, 0, \chi) = \epsilon^s(N_0)$. □

Next, we prove Proposition 9.

Proof of Proposition 9: Differentiating (IA8) and simplifying yields

$$\frac{\partial p_1^h(e^{\mu(\theta)x}N_0)}{\partial x} = \frac{\mu(\theta)p_1^h(e^{\mu(\theta)x}N_0)}{\epsilon^s(e^{\mu(\theta)x}N_0) + \epsilon}.$$

Using this equation, we differentiate (IA9) with respect to x and simplify to obtain

$$\begin{aligned}\frac{\partial \log p_0^h(N_0, x, z, \chi)}{\partial x} = {} & \frac{c_1 p_1^h(e^{\mu_d^{max}x}N_0)}{p_0^h(N_0, x, z, \chi)} \frac{\mu_d^{max}}{\epsilon^s(e^{\mu_d^{max}x}N_0) + \epsilon} + \frac{c_2 p_1^h(e^{\mu_r^{max}x}N_0)}{p_0^h(N_0, x, z, \chi)} \frac{\mu_r^{max}}{\epsilon^s(e^{\mu_r^{max}x}N_0) + \epsilon} \\ & + \int_\Theta \frac{c_3 p_1^h(e^{\mu(\theta)x}N_0)}{p_0^h(N_0, x, z, \chi)} \frac{\mu(\theta)}{\epsilon^s(e^{\mu(\theta)x}N_0) + \epsilon} f_r(\theta) d\theta,\end{aligned}$$

where $c_i = \gamma_i/(\gamma_1 + \gamma_2 + \gamma_3)$ for $i \in \{1, 2, 3\}$ and the γ_i are defined as follows:

$$\begin{aligned}\gamma_1 &= (D^l)'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_d^{max}x}N_0)) \\ \gamma_2 &= \chi N_0 D'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max}x}N_0)) \\ \gamma_3 &= (1 - \chi)N_0 \int_\Theta D'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta) d\theta.\end{aligned}$$

We now prove the equations in the proposition. When $x = 0$, $r_0^h(N_0, x, z, \chi) = p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu(\theta)x}N_0)$ for all θ, so $\gamma_1 = \epsilon^s(N_0)/(\epsilon^s(N_0) + \epsilon)$, $\gamma_2 = \chi\epsilon/(\epsilon^s(N_0) + \epsilon)$, and $\gamma_3 = (1 - \chi)\epsilon/(\epsilon^s(N_0) + \epsilon)$. We also have $p_0^h(N_0, 0, z, \chi) = p_1^h(N_0)/2$. It follows that

$$\frac{\partial \log p_0^h(N_0, 0, z, \chi)}{\partial x} = \frac{1}{2} \frac{\epsilon^s(N_0)\mu_d^{max} + \chi\epsilon\mu_r^{max} + (1 - \chi)\epsilon\overline{\mu}}{\epsilon^s(N_0) + \epsilon} \frac{1}{\epsilon^s(N_0) + \epsilon},$$

which coincides with the formula in the text. When $z = 0$, $\mu(\theta) = \overline{\mu}$ for all θ and $p_0^h(N_0, x, z, \chi) = p_1^h(N_0) + p_1^h(e^{\overline{\mu}x}N_0)$ as shown in the previous proof. It follows that

$$\frac{\partial \log p_0^h(N_0, x, 0, \chi)}{\partial x} = \frac{p_1^h(e^{\overline{\mu}x}N_0)}{p_1^h(N_0) + p_1^h(e^{\overline{\mu}x}N_0)} \frac{\overline{\mu}}{\epsilon^s(e^{\overline{\mu}x}N_0) + \epsilon}.$$

This expression coincides with the formula in the text because

$$\frac{p_1^h(e^{\overline{\mu}x}N_0)}{p_1^h(N_0)} = \exp\left(\int_0^x \frac{\partial \log p_1^h(e^{\overline{\mu}x'}N_0)}{\partial x'} dx'\right) = \exp\left(\int_0^x \frac{\overline{\mu}dx'}{\epsilon^s(e^{\overline{\mu}x'}N_0) + \epsilon}\right).$$

□

Figure IA1 plots the $t = 0$ pass-through $1/(\epsilon^s(N_0) + \epsilon)$ and $t = 1$ pass-through

$1/(\epsilon^s(e^{\overline{\mu}x}N_0)+\epsilon)$ as well as the approximation for $\partial \log p_0^h(N_0,x,z,\chi)/\partial x$ with and without disagreement. Disagreement amplifies the price impact of x most when the short-run elasticity is high and the long-run elasticity is low.

IV. Supplements to Section V.C

Pulte Investor Presentation. Figure IA2 presents slides from a 2004 presentation to investors by Pulte, one of the large public homebuilders studied in Section V.B. These slides provide some evidence that builders viewed supply constraints as binding in the long run across many cities during the housing boom, and also that our partition of cities in Figure 5 matches that considered by builders contemporaneously with the boom.

Construction Analysis. To analyze the effect of the shock x on construction, we define $Q_r(N_0,x,z)$ to be the quantity of housing held by potential residents at $t=0$ in equilibrium. The following lemma characterizes the response of Q_r to x.

LEMMA IA9: *$Q_r(N_0,x,z) < Q_r(N_0,0,z)$ if $e^{-\mu_d^{max}x} < N_0 < N_0^*(x,z)$ and $z=0$. $Q_r(N_0,x,z) = Q_r(N_0,0,z)$ otherwise.*

Proof: By Proposition 2, $Q_r(N_0,x,z)=1$ when $N_0 \geq N_0^*(x,z)$. By (A1) in the proof of Lemma 2, $Q_r(N_0,x,z) = SN_0\int_\Theta D(p_1^h(e^{\mu_d^{max}x}N_0)+k-p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta$ when $N_0 < N_0^*(x,z)$.

When $z=0$, $p_1^h(e^{\mu_d^{max}x}N_0) = p_1^h(e^{\mu(\theta)x}N_0)$ for all $\theta\in\Theta$, so $Q_r(N_0,x,0)=SN_0$ for $N_0 < N_0^*(x,0)$. Because $N_0^*(x,0)=1$ as shown by Proposition 1, $Q_r(N_0,x,0)=Q_r(N_0,0,0)$.

When $z>0$ and $N_0 \leq e^{-\mu_d^{max}x}$, $p_1^h(e^{\mu(\theta)x}N_0) \geq p_1^h(e^{\mu_d^{max}x}N_0)$ for all $\theta\in\Theta$, so by Assumption 1 $Q_r(N_0,x,z)=SN_0$. Thus, $Q_r(N_0,x,z)=Q_r(N_0,0,z)$ in this case as well.

When $z>0$ and $N_0 \geq N_0^*(x,z)$, $Q_r(N_0,x,z)=1$ and $Q_r(N_0,x,0)=1$ because $N_0 \geq N_0^*(x,z) > N_0^*(x,0)=1$. Again, $Q_r(N_0,x,z)=Q_r(N_0,0,z)$ in this case.

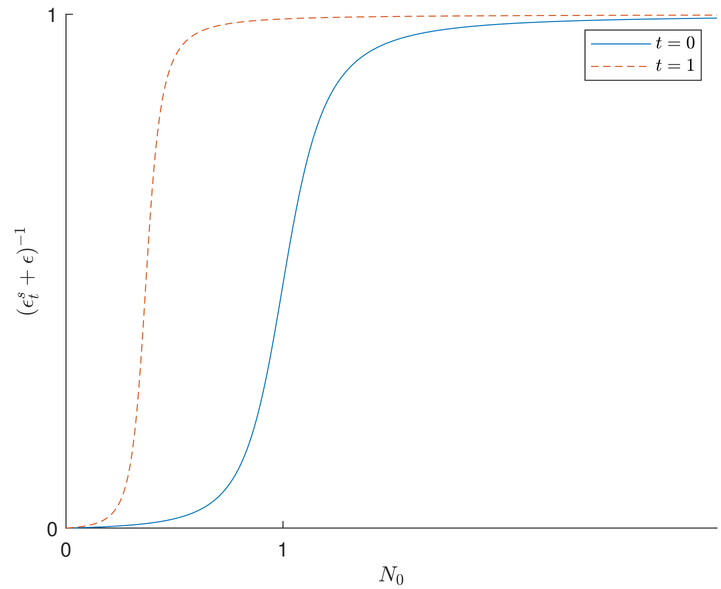
We divide the final case in which $e^{-\mu_d^{max}x} < N_0 < N_0^*(x,z)$ and $z>0$ into two subcases. If $1 \leq N_0 < N_0^*(x,z)$, then $1 = N_0^*(0,z) \leq N_0 < N_0^*(x,z)$. It follows that $Q_r(N_0,x,z) < 1 = Q_r(N_0,0,z)$, as claimed. If $e^{-\mu_d^{max}x} < N_0 < 1$, then $Q_r(N_0,0,z)-Q_r(N_0,x,z) = SN_0\int_{\theta<\theta_d^{max}}(1-D(p_1^h(e^{\mu_d^{max}x}N_0)+k-p_1^h(e^{\mu(\theta)x}N_0)))f_r(\theta)d\theta$. For all $\theta<\theta_d^{max}$, the integrand is positive because $e^{\mu_d^{max}x}N_0>1$. By Assumption 4, $\int_{\theta<\theta_d^{max}} f_r(\theta)d\theta > 0$, so $Q_r(N_0,0,z) > Q_r(N_0,x,z)$, as claimed. $\square$

Lemma IA9 shows that the shock x only affects the equilibrium quantity of housing in intermediate cities with disagreement, in which case the shock lowers the housing stock. Because the shock does not change the current demand N_0, it does not alter housing supply in most cases. It only does so when optimistic developers set prices so high that the number of potential residents choosing to buy falls. This scenario occurs in intermediate cities with disagreement.

References to Internet Appendix

Saiz, Albert, 2010, The Geographic Determinants of Housing Supply, *Quarterly Journal of Economics* 125, 1253–1296.

Panel A. Modified Inverse Supply Elasticity



Panel B. Price Increase

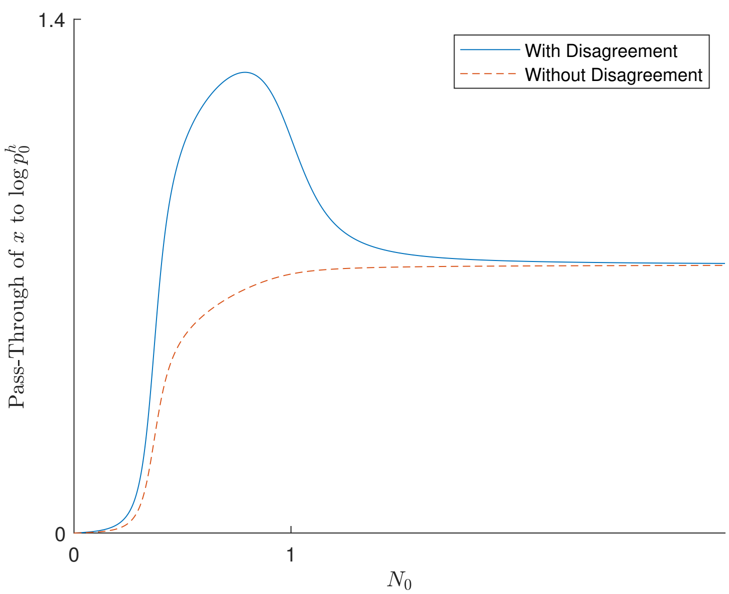


Figure IA1. Comparative statics with respect to initial demand. This figure plots modified inverse supply elasticities and the sensitivity of the price of housing to the shock x for different values of N_0, the number of potential residents at $t = 0$ relative to the city size. The modified inverse supply elasticity equals $1/(\epsilon_t^s + \epsilon)$, where ϵ is the absolute elasticity of the demand function $D(\cdot)$ and ϵ_t^s is the elasticity of supply at time t under the belief without disagreement, $\overline{\mu}$. These supply elasticities equal $\epsilon_0^s = \epsilon^s(N_0)$ and $\epsilon_1^s = \epsilon^s(e^{\overline{\mu}x} N_0)$; $\epsilon^s(\cdot)$ is defined by Lemma IA8. The pass-through of x to the log equilibrium house price at $t = 0$, $\log p_0^h$, equals the expression for $\partial \log p_0^h(N_0, x, z, \chi)/\partial x$ given by Proposition 9. The parameters used to generate this figure are $k = 1$, $x = 1$, $z = 1$, $\epsilon = 1$, $\chi = 0$, $\overline{\mu} = 1$, $f_r = f_d = (0.9)\mathbf{1}_{-1/9} + (0.1)\mathbf{1}_1$, and $D^l(r) = 0.01k/r$, with $z = 0$ used in the "without disagreement" graph.

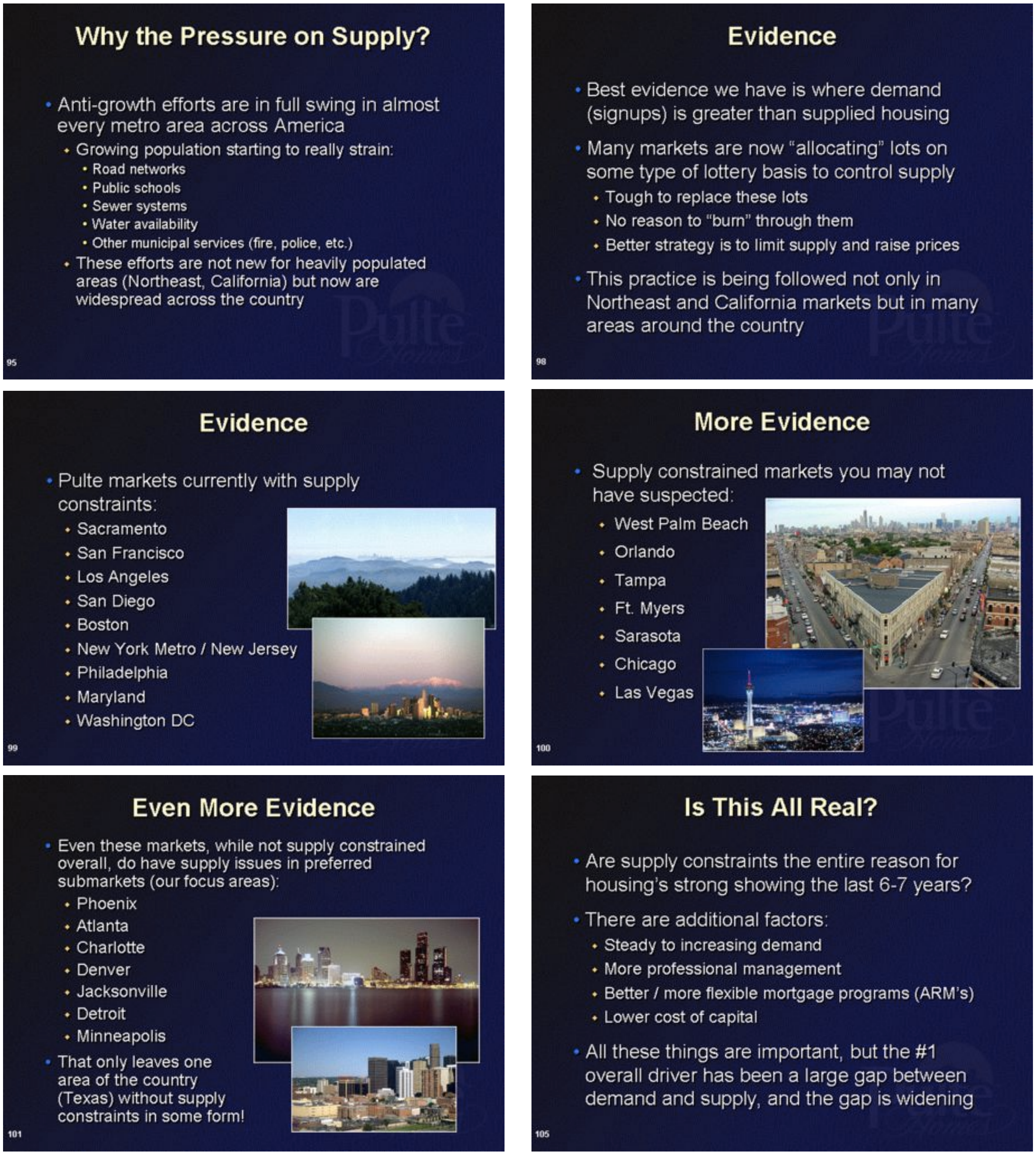


Figure IA2. Land supply slides from Pulte's 2004 investor conference. This figure provides slides excerpted from a presentation by Pulte Homes, Inc., on February 26, 2004, to investors and disclosed under SEC Regulation FD requirements. Last accessed on March 15, 2015, at `http://services.corporate-ir.net/SEC.Enhanced/SecCapsule.aspx?c=77968&fid=2633894`.

Table IAI
Annualized Real House Price Growth, 2000 to 2006

This table presents estimates of α and β from the equation

$$\Delta \log p_j^h = \alpha 1_{j \in \{\text{Anomalous Cities}\}} + \frac{\beta \mathbf{d}_j + \eta_j}{\epsilon_j^s + \epsilon},$$

where j indexes metropolitan areas and $\Delta \log p_j^h$ equals the 2000 to 2006 annualized log change in the second-quarter FHFA house price index deflated by the CPI-U. The Anomalous Cities are metro areas in our sample in Arizona, inland California, Florida, and Nevada. In specification (1), $\mathbf{d}_j$ includes just a constant; in specification (2), $\mathbf{d}_j$ further includes the listed demographics from the 2000 U.S. Census, which are measured in shares of the population except for log population and log income. We set $\epsilon = 0.6$ and take the housing supply elasticity ϵ_j^s from Saiz (2010). We estimate α and β by multiplying each side of the above equation by $\epsilon_j^s + \epsilon$ and then performing OLS with η_j as the error term. Standard errors are in parentheses. Significance levels 10%, 5%, and 1% are denoted respectively by *, **, and ***.

	(1)	(2)
Anomalous City	0.077***	0.066***
	(0.0082)	(0.0080)
Elasticity-adjusted demand controls		
Log population		-0.0083
		(0.0073)
Log income		0.11
		(0.091)
White		-0.15
		(0.10)
White, not hispanic		0.0050
		(0.082)
Less than 9th grade		0.38
		(0.28)
9-12th grade, no diploma		-0.25
		(0.32)
Unemployment		1.66**
		(0.58)
Poverty		-0.57
		(0.60)
Constant	0.080***	-0.87
	(0.0062)	(1.05)
Observations	105	105
R^2	0.46	0.68